1.10

Lesson ~~4.11~~ - Sketching a Polynomial Graph

Learning Objectives: Students will be able to:

1. Create a rough sketch of the graph of a polynomial given its equation

Making a connection

- We learned some key characteristics about polynomial graphs in lessons 4.9 and 4.10. In this lesson, we will use these characteristics to create a rough sketch of a polynomial graph.
- This page will model one problem and explain the process. The graph on the next page is the completed graph with comments

Example: Sketch the graph of the polynomial $\mathbf{x^3 + 6x^2 - 9x - 54}$

Before we start graphing, the following is a "check list" of information that we need to construct our graph as well as how to determine each:

1. Identify the graph's end behavior

> In lesson 4.9 we learned how to determine whether the graph "rises/falls" either "right/left" by looking at the sign of the highest degree turn and whether it is even or odd. (you need to know the chart contained in lesson 4.9)

> In this example, the highest degree is odd and the sign of the highest degree is positive. Therefore, the end behavior of the graph as it goes to infinity will be **"rises left and falls right"** (the arrows will point in these directions).

> **Tip** - Before you construct your actual graph, write the end behavior after you identify it but do not complete the graph until your checklist is complete

2. Identify the Maximum number of turns that the graph can make

> Also in lesson 4.9, we learned that the maximum number of turns that a polynomial graph can make is "one less than its degree".

> In this example the degree of the polynomial is "3" so **the maximum number of turns that it will make is 2**.

> When we do our graphs we will always graph the maximum number of turns

> **Note** - We will NOT be able to determine the exact coordinate of each turning point (the max or min). When you do your graph, you will NOT have to identify this point, you will only need to show the turn

3. Identify the "zeros" and multiplicity of the polynomial's factors

> In lesson 4.10, we learned how to find a polynomial's zeros by factoring and setting the factors equal to zero

> In lesson 4.10 we also learned how to identify a factor's multiplicity by looking at the power to which the factor is being raised. Depending one whether the multiplicity is 1, 2 odd or even, we can identify whether the graph will "bounce", "pass" or "wiggle" through the x-axis when it touches its zero

> If the polynomial you are given is not factored, you must factor it.

> In this example, since the polynomial has four terms we would factor by grouping:

$(x^3 + 6x^2)\ (-9x - 54)$

$x^2 (x + 6) -9 (x + 6)$

$(x^2 - 9)(x + 6)$ ----------> **note $x^2 - 9$ is the diff of 2 sqrs**

$(x + 3)(x - 3)(x + 6)$

> Based on these factors, **the coordinates of the zeros will be (-3, 0), (6, 0), (-6, 0)**

> Since each factor has multiplicity 1, **the graph will pass through each zero**

> **Note** - Remember, factors with multiplicity 2 or greater, still only yield one zero

4. Identify the "y-intercept" of the graph

> As with both linear and quadratic equations, the "constant" term represents the point on the graph where it crosses the "y" intercept (x = 0)

> In this example, the constant is -54, therefore **the coordinate of the y intercept would be (0, -54)**

> **Note** - if the polynomial you are given does NOT have a constant, then the y intercept is (0, 0)

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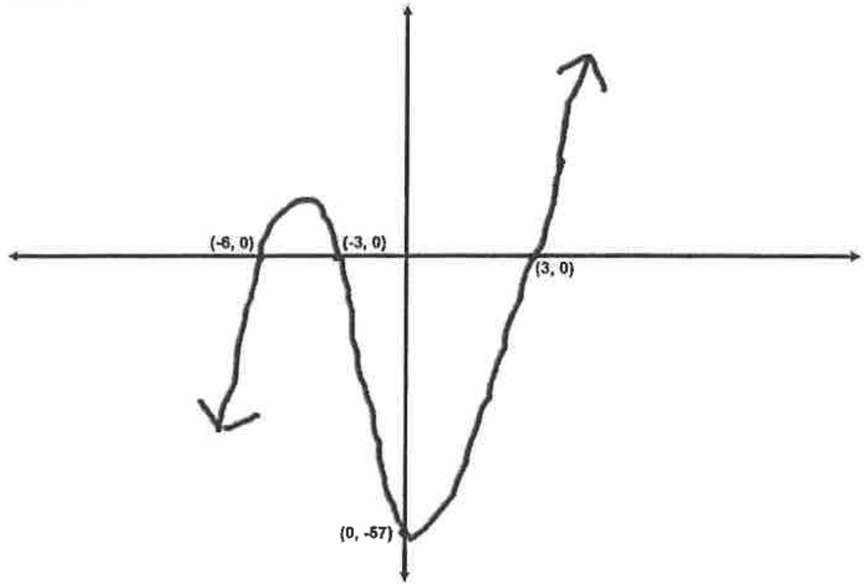
Lesson ~~[illegible]~~ - Sketching a Polynomial Graph

Constructing the graph

- Recapping from our example we know that our graph:
  - > Will have end behavior that falls left and rises right as it goes to infinity
  - > Will have two turns
  - > Will pass through zeros of (-3, 0), (6, 0), (-6, 0), all have multiplicity 1
  - > Will cross the x axis at (0, -54)

- Put all of the information above on your graph, and use the end behavior to draw your graph
  - > **Note** - you will not know the exact max/min points, but you will know the number of turns



Your Turn Sketch the graph of the polynomial: $\mathbf{x^3 + 3x^2 - 16x - 48}$

- End Behavior

Fall left, rise right

- Max # of Turns

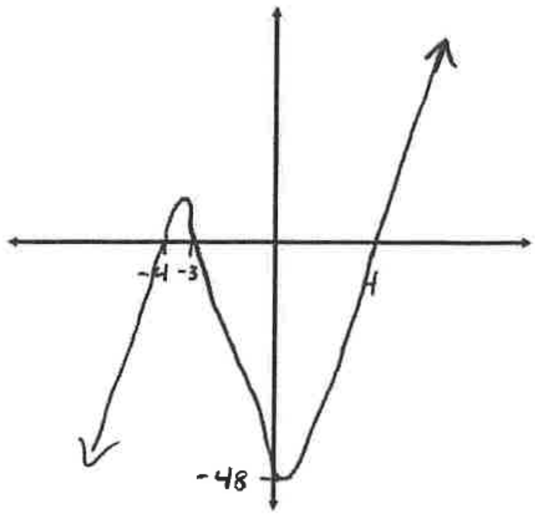
2

- Factor the Polynomial

$(x^3+3x^2)\ (-16x-48)$

$x^2(x+3) - 16\ (x+3)$

$(x^2-16)(x+3) \rightarrow \boxed{(x+4)\ (x-4)\ (x+3)}$



- Zeros

$x = -4,\ x = 4,\ x = -3$

- "Pass", "Bounce" or "wiggle" thru each zero

pass thru all, all have multiplicity 1

- Y-Intercept

$Y = 0, -48$

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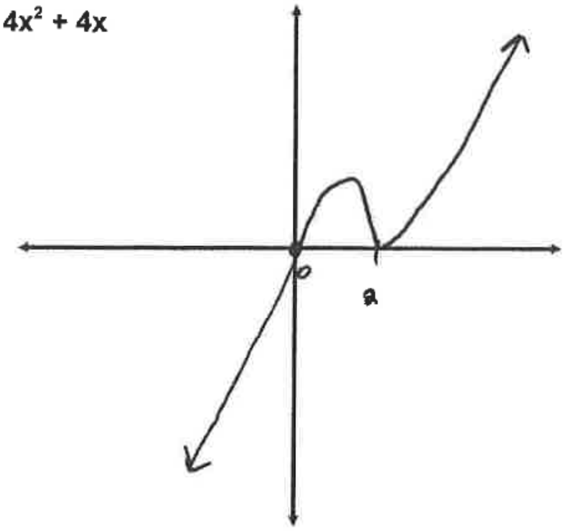
Lesson - Sketching a Polynomial Graph

Practice

1) Sketch the graph of the polynomial: $x^3 - 4x^2 + 4x$



- End Behavior

Fall left, rise right

- Max # of Turns

2

- Factor the Polynomial

$x(x^2-4x+4)$

$x(x-2)(x-2)$

$\boxed{x(x-2)^2}$

- Zeros

$x=0$, $x=+2$

- "Pass", "Bounce" or "wiggle" thru each zero

Pass through $x=0$, bounce off $x=2$

- Y-Intercept

$(0,0)$

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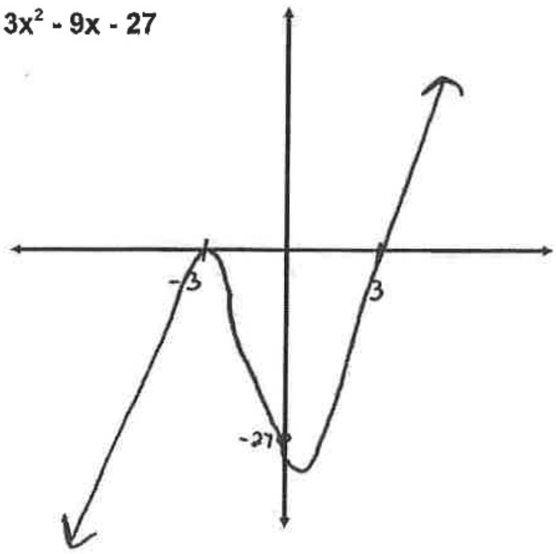
2) Sketch the graph of the polynomial: $x^3 + 3x^2 - 9x - 27$



- End Behavior

Fall left, rise right

- Max # of Turns

2

- Factor the Polynomial

$(x^3+3x^2)(-9x-27)$

$x^2(x+3)-9(x+3)$

$(x+3)(x^2-9) \rightarrow (x+3)(x+3)(x-3)$

$\boxed{(x+3)^2(x-3)}$

- Zeros

$x=3$, $x=-3$

- "Pass", "Bounce" or "wiggle" thru each zero

bounce of $x=-3$, pass thru $x=3$

- Y-Intercept

$(0,-27$

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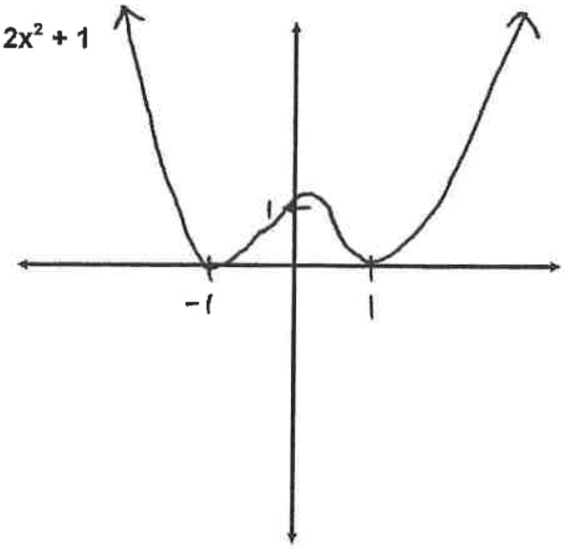
Lesson [illegible] - Sketching a Polynomial Graph

Practice

3) Sketch the graph of the polynomial: $x^4 - 2x^2 + 1$



- End Behavior

rise left, rise right

- Max # of Turns

3

- Factor the Polynomial

$(x^2-1)(x^2-1)$

$(x+1)(x-1)\ (x+1)(x-1)$

$\boxed{(x+1)^2\ (x-1)^2}$

- Zeros

$x=1,\ x=-1$

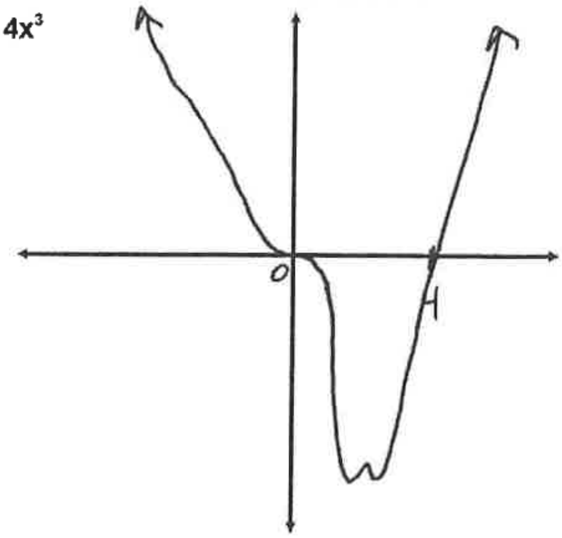
- "Pass", "Bounce" or "wiggle" thru each zero

Bounce off both (multiplicity 2)

- Y-Intercept

(0,1)

4) Sketch the graph of the polynomial: $x^4 - 4x^3$



- End Behavior

rise left, rise right

- Max # of Turns

3

- Factor the Polynomial

GCF $\boxed{x^3(x-4)}$

- Zeros

$x=0,\ x=4$

- "Pass", "Bounce" or "wiggle" thru each zero

wiggle thru $x=0$ (multiplicity 3) pass thru $x=4$

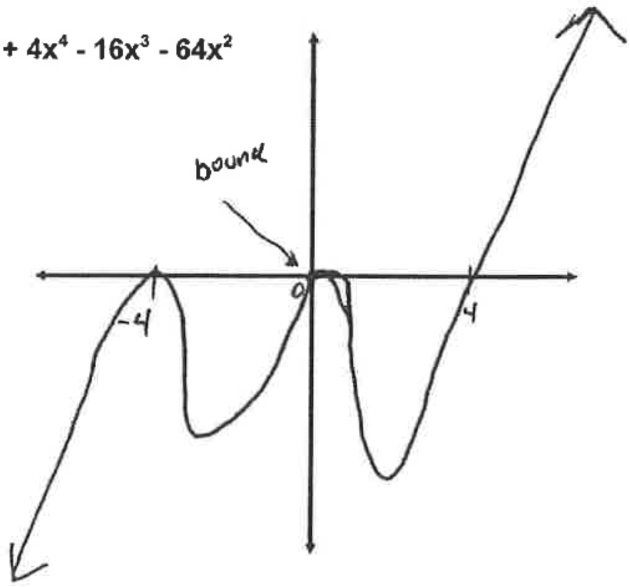
- Y-Intercept

(0,0)

Lesson ~~4.11~~ 1.10 - Sketching a Polynomial Graph

Practice

5) Sketch the graph of the polynomial: $x^5 + 4x^4 - 16x^3 - 64x^2$



- End Behavior

Fall left, rise right

- Max # of Turns

4

- Factor the Polynomial

$x^2(x^3+4x^2-16x-64)$

$x^2((x^3+4x^2)(-16x-64)$

$x^2(x^2(x+4)-16(x+4))$

$x^2(x+4)(x^2-16) \to x^2(x+4)(x+4)(x-4)$

$x^2(x+4)^2(x-4)$

- Zeros

$x=0,\ x=-4,\ x=4$

- "Pass", "Bounce" or "wiggle" thru each zero

bounce off $x=0$ and $x=-4$, pass thru $x=4$

- Y-Intercept

$(0,0)$

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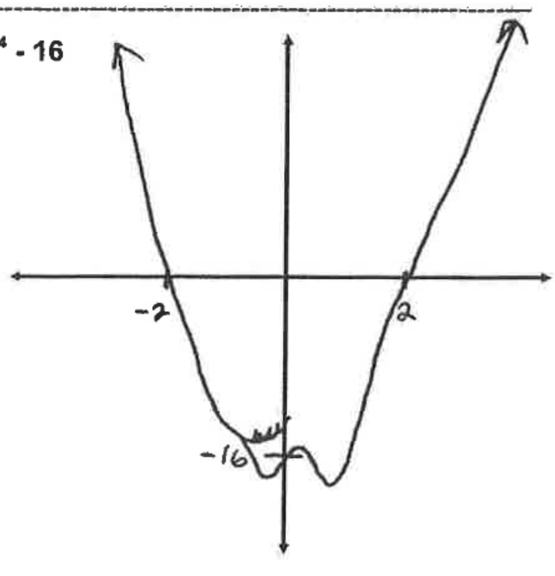
6) Sketch the graph of the polynomial: $x^4 - 16$



- End Behavior

rise left, rise right

- Max # of Turns

3

- Factor the Polynomial

$(x^2-4)\ (x^2+4)$

$(x+4)(x-4)\ (x^2+4)$ ↳ $\boxed{\text{No, zero, would be complex \# if solved}}$

- Zeros

$x=4,\ x=-4$ only

- "Pass", "Bounce" or "wiggle" thru each zero

Pass thru both $x=0 \to$ have multiplicity 1

- Y-Intercept

$(0,-16)$

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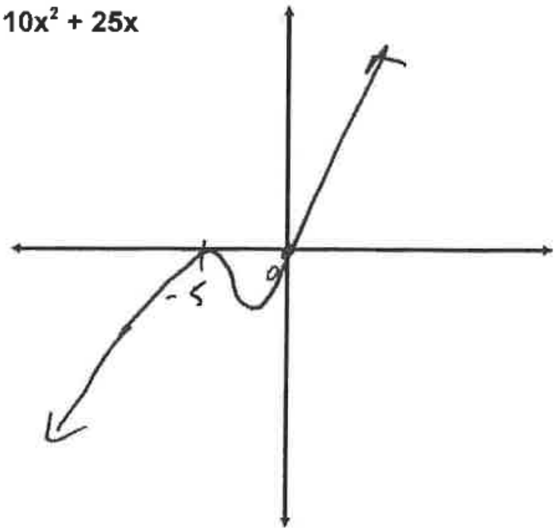
Lesson ~~4.11~~ - Sketching a Polynomial Graph

Practice

7) Sketch the graph of the polynomial: $x^3 + 10x^2 + 25x$



- End Behavior

Fall left rise right

- Max # of Turns

2

- Factor the Polynomial

$x\,(x^2+10x+25$

$x\,(x+5)(x+5)$

$x\,(x+5)^2$

- Zeros

$x=0$, $x=-5$

- "Pass", "Bounce" or "wiggle" thru each zero

Pass thru $x=0$, bounce off $x=-5$

- Y-Intercept

(0,0)

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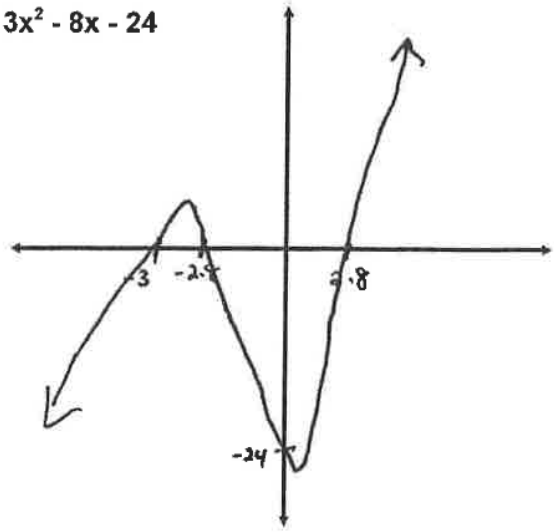
8) Sketch the graph of the polynomial: $x^3 + 3x^2 - 8x - 24$



- End Behavior

Fall left, rise right

- Max # of Turns

2

- Factor the Polynomial

$(x^3+3x^2)\ (-8x-24)$

$x^2(x+3)-8(x+3)$

$(x^2-8)\ (x+3)$ → zeros = $\pm\sqrt{8} = \pm 2.8$

- Zeros

$x=-3$, $x=+2.8$ $x=-2.8$

- "Pass", "Bounce" or "wiggle" thru each zero

Pass thru all 3 → multiplicity 1

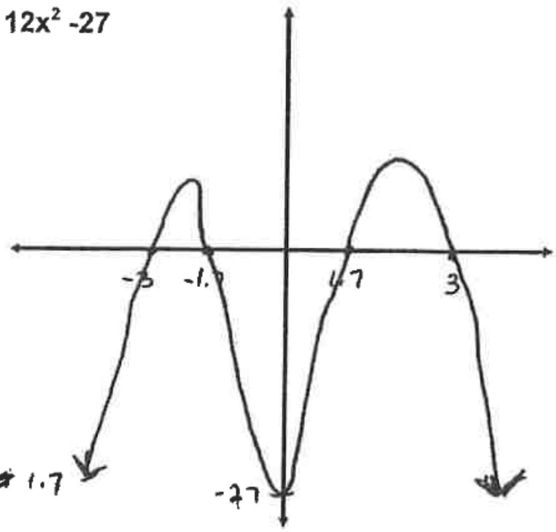
- Y-Intercept

(0, -24)

Lesson 1.10 - Sketching a Polynomial Graph

Practice

9) Sketch the graph of the polynomial: $-x^4 + 12x^2 - 27$



- End Behavior

Fall left, Fall right

- Max # of Turns

3

- Factor the Polynomial

$-1\,(x^4 - 12x^2 + 27)$

$-1\,(x^2 - 9)(x^2 + 3)$

$-1\,(x+3)(x-3)(x^2+3)$

$\hookrightarrow$ zeros $= \pm\sqrt{3} = -1.7,\ 1.7$

- Zeros

$x = 3,\ x = -1.7,\ x = 1.7$

- "Pass", "Bounce" or "wiggle" thru each zero

Pass thru all 3 → multiplicity 1

- Y-Intercept

$(0, -27)$

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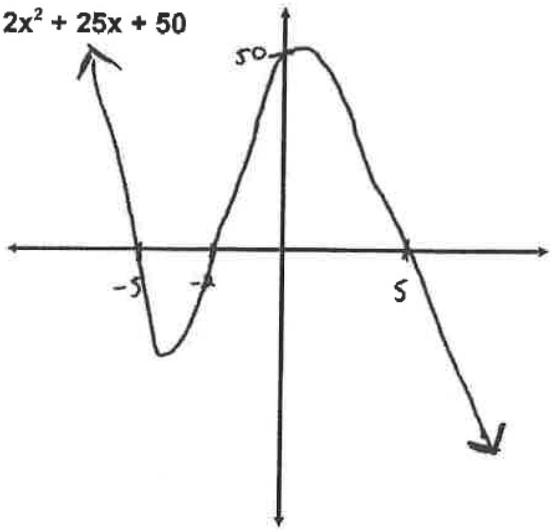
10) Sketch the graph of the polynomial: $-x^3 - 2x^2 + 25x + 50$



- End Behavior

rise right, fall left

- Max # of Turns

2

- Factor the Polynomial

$-1\,(x^3 + 2x^2 - 25x - 50)$

$-1\,(x^3 + 2x^2)(-25x - 50)$

$-1\,(x^2(x+2) - 25(x+2))$ $\quad -1\,(x+2)(x+5)(x-5)$

$-1\,(x+2)(x^2-25) \rightarrow$

- Zeros

$x = -2,\ x = -5,\ x = 5$

- "Pass", "Bounce" or "wiggle" thru each zero

Pass thru all 3

- Y-Intercept

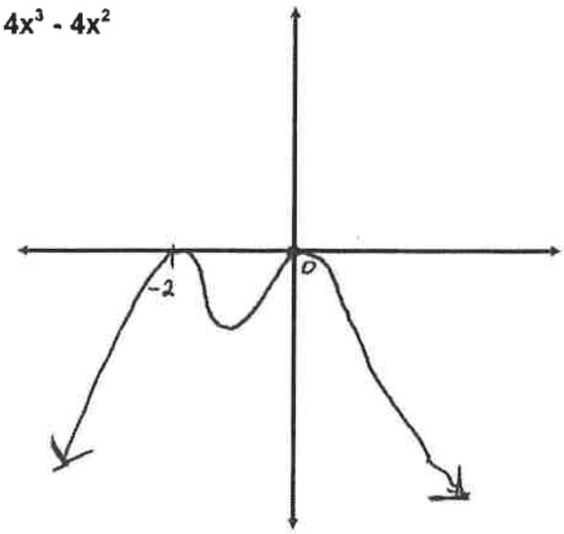
$(0, 50)$

1.10

Lesson [redacted] - Sketching a Polynomial Graph

Practice

11) Sketch the graph of the polynomial: $-x^4 - 4x^3 - 4x^2$



- End Behavior

Fall left, fall right

- Max # of Turns

3

- Factor the Polynomial

$-1 \ (x^4 + 4x^3 + 4x^2$

$-1 \cdot x^2 \ (x^2 + 4x + 4)$

$-1 \ x^2 \ (x+2)(x+2) \rightarrow -1 \ x^2 (x+2)^2$

- Zeros

$x = 0, \ x = -2$

- "Pass", "Bounce" or "wiggle" thru each zero

Bounce of both → multiplicity 2

- Y-Intercept

$(0, 0)$

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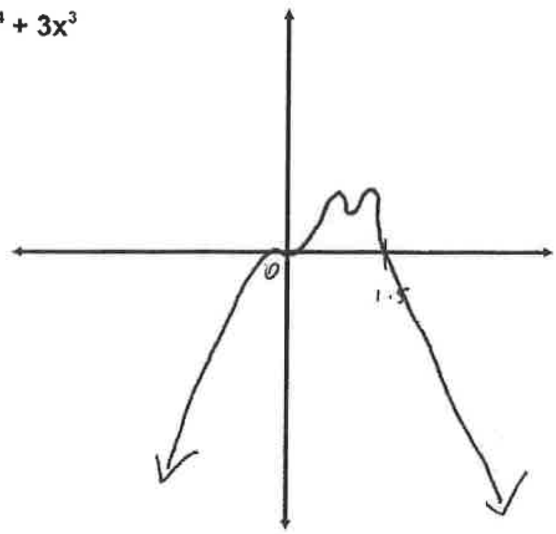
12) Sketch the graph of the polynomial: $-2x^4 + 3x^3$



- End Behavior

Fall left, fall right

- Max # of Turns

3

- Factor the Polynomial

$-1 \ x^3 \ (2x - 3)$

- Zeros

$x = 0 \ , \ x = \frac{3}{2}$ or 1.5

- "Pass", "Bounce" or "wiggle" thru each zero

wiggle thru $x = 0$, pass thru $x = \frac{3}{2}$

- Y-Intercept

$(0, 0)$