

Lesson 1.11 - Writing polynomial equations (real zeros)

Learning Objectives: SWBAT

- Write the equation of a polynomial function given its real zeros and multiplicity

Making a connection

- In lesson 1.9 we found all of the real zeros of a polynomial function equation by using factors. From the factors, we also determined a zero's multiplicity
- In this lesson we will be working "backwards" to create function equations by using real zeros and multiplicity to create (and then multiply) factors

Examples: Write the equations of the following functions given the zeros:

a. $-\frac{1}{2}, 3, 3$ b. $3, 2 + \sqrt{11}, 2 - \sqrt{11}$

Solution

- a. Note that the zero $x = -\frac{1}{2}$ corresponds to either $(x + \frac{1}{2})$ or $(2x + 1)$. To avoid fractions, choose the second factor and write

$$\begin{aligned} f(x) &= (2x + 1)(x - 3)^2 \\ &= (2x + 1)(x^2 - 6x + 9) = 2x^3 - 11x^2 + 12x + 9. \end{aligned}$$

- b. For each of the given zeros, form a corresponding factor and write

$$\begin{aligned} f(x) &= (x - 3)[x - (2 + \sqrt{11})][x - (2 - \sqrt{11})] \\ &= (x - 3)[(x - 2) - \sqrt{11}][(x - 2) + \sqrt{11}] \\ &= (x - 3)[(x - 2)^2 - (\sqrt{11})^2] \\ &= (x - 3)(x^2 - 4x + 4 - 11) \\ &= (x - 3)(x^2 - 4x - 7) = x^3 - 7x^2 + 5x + 21. \end{aligned}$$

Your Turn: Write the equation of the following functions given the zeros:

$-1, 2 + \sqrt{5}, 2 - \sqrt{5}$

Factors: $(x+1) (x - (2 + \sqrt{5})) (x - (2 - \sqrt{5}))$

$$(x+1) ((x-2) - \sqrt{5}) ((x-2) + \sqrt{5})$$

$$(x+1) ((x-2)^2 - (\sqrt{5})^2)$$

$$(x+1) (x^2 + 4x + 4 - 5)$$

$$(x+1) (x^2 + 4x - 1)$$

$$x^3 + 4x^2 - x + x^2 + 4x - 1$$

$$\boxed{x^3 + 5x^2 + 3x - 1}$$

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Practice: Write the equations of the following functions given the zeros:

49. 0, 4

$$f(x) = x(x-4)$$

$$f(x) = x^2 - 4x$$

51. 0, -2, -3

$$f(x) = x(x+2)(x+3)$$

$$f(x) = x(x^2 + 5x + 6)$$

$$f(x) = x^3 + 5x^2 + 6x$$

53. 4, -3, 3, 0

$$f(x) = x(x-4)(x+3)(x-3)$$

$$f(x) = x(x-4)(x^2-9)$$

$$f(x) = x(x^3 - 4x^2 - 9x + 36)$$

$$f(x) = x^4 - 4x^3 - 9x^2 + 36x$$

55. $1 + \sqrt{3}, 1 - \sqrt{3}$

$$f(x) = (x - (1 + \sqrt{3}))(x - (1 - \sqrt{3}))$$

$$f(x) = ((x-1) + \sqrt{3})((x-1) - \sqrt{3})$$

$$f(x) = (x-1)^2 - (\sqrt{3})^2$$

$$f(x) = x^2 - 2x + 1 - 3 = x^2 - 2x - 2$$

57. 2, $4 + \sqrt{5}, 4 - \sqrt{5}$

$$f(x) = (x-2)((x-(4+\sqrt{5}))((x-(4-\sqrt{5})))$$

$$f(x) = (x-2)((x-4) + \sqrt{5})((x-4) - \sqrt{5})$$

$$f(x) = (x-2)((x-4)^2 - (\sqrt{5})^2)$$

$$f(x) = (x-2)(x^2 - 8x + 16 - 5)$$

$$f(x) = (x-2)(x^2 - 8x + 11)$$

$$f(x) = x^3 - 8x^2 + 11x - 2x^2 + 16x - 22$$

$$f(x) = x^3 - 10x^2 + 27x - 22$$

50. -7, 2

$$f(x) = (x+7)(x-2)$$

$$f(x) = x^2 + 5x - 14$$

52. 0, 2, 5

$$f(x) = x(x-2)(x-5)$$

$$f(x) = x$$

$$f(x) = x(x^2 - 7x + 10)$$

$$f(x) = x^3 + 7x^2 + 10x$$

54. -2, -1, 0, 1, 2

$$f(x) = x(x+1)(x-1)(x+2)(x-2)$$

$$f(x) = x(x^2-1)(x^2-4)$$

$$f(x) = x(x^4 - 5x^2 + 4)$$

$$f(x) = x^5 - 5x^3 + 4x$$

56. $6 + \sqrt{3}, 6 - \sqrt{3}$

$$f(x) = ((x - (6 + \sqrt{3}))((x - (6 - \sqrt{3})))$$

$$f(x) = ((x-6) + \sqrt{3})((x-6) - \sqrt{3})$$

$$f(x) = (x-6)^2 - (\sqrt{3})^2$$

$$f(x) = x^2 + 12x + 36 - 3 = x^2 + 12x + 33$$

58. 4, $2 + \sqrt{7}, 2 - \sqrt{7}$

$$f(x) = (x-4)((x-(2+\sqrt{7}))((x-(2-\sqrt{7})))$$

$$f(x) = (x-4)((x-2) + \sqrt{7})((x-2) - \sqrt{7})$$

$$f(x) = (x-4)((x-2)^2 - (\sqrt{7})^2)$$

$$f(x) = (x-4)(x^2 - 4x + 4 - 7)$$

$$f(x) = (x-4)(x^2 - 4x - 3)$$

$$f(x) = x^3 - 4x^2 - 3x - 4x^2 + 16x + 12$$

$$f(x) = x^3 - 8x^2 + 13x + 12$$

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Practice: Write the equations of the following functions given the zeros/multiplicity:

59. Zero: -2, multiplicity: 2

Zero: -1, multiplicity: 1

$$f(x) = (x+2)^2 (x+1)$$

$$f(x) = (x^2 + 4x + 4)(x+1)$$

$$f(x) = x^3 + 4x^2 + 4x + x^2 + 4x + 4$$

$$f(x) = x^3 + 5x^2 + 8x + 4$$

60. Zero: 3, multiplicity: 1

Zero: 2, multiplicity: 3

$$f(x) = (x-3)(x-2)^3$$

$$f(x) = (x-3)(x-2)(x-2)(x-2)$$

$$f(x) = (x^2 - 5x + 6)(x^2 - 4x + 4)$$

$$f(x) = x^4 - 9x^3 + 30x^2 - 44x + 24$$

61. Zero: -4, multiplicity: 2

Zero: 3, multiplicity: 2

$$f(x) = (x+4)^2 (x-3)^2$$

$$f(x) = (x^2 + 8x + 16)(x^2 - 6x + 9)$$

$$f(x) = x^4 + 2x^3 - 23x^2 - 24x + 144$$

62. Zero: -5, multiplicity: 3

Zero: 0, multiplicity: 2

$$f(x) = x^2 (x+5)^3$$

$$f(x) = x^2 (x+5)(x+5)(x+5)$$

$$f(x) = (x^3 + 5x^2)(x^2 + 10x + 25)$$

$$f(x) = x^5 + 15x^4 + 75x^3 + 125x^2$$

63. Zero: -1, multiplicity: 2

Zero: -2, multiplicity: 1

$$f(x) = (x+1)^2 (x+2)$$

$$f(x) = (x^2 + 2x + 1)(x+2)$$

$$f(x) = x^3 + 4x^2 + 5x + 2$$

64. Zero: -1, multiplicity: 2

Zero: 4, multiplicity: 2

$$f(x) = (x+1)^2 (x-4)^2$$

$$f(x) = (x^2 + 2x + 1)(x^2 - 8x + 16)$$

$$f(x) = x^4 - 6x^3 + x^2 + 24x + 16$$

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Practice: Write the equations of the following functions given the zeros/multiplicity:

C) $\pm 1, \pm\sqrt{2}$

$$f(x) = (x+1)(x-1)(x+\sqrt{2})(x-\sqrt{2})$$

$$f(x) = (x^2-1)(x^2-2)$$

$$\boxed{f(x) = x^4 - 3x^2 + 2}$$

F) $\pm 4, 0, \pm\sqrt{2}$

$$f(x) = x(x+4)(x-4)(x+\sqrt{2})(x-\sqrt{2})$$

$$f(x) = x(x^2-16)(x^2-2)$$

$$f(x) = x(x^4 - 18x^2 + 32)$$

$$\boxed{f(x) = x^5 - 18x^3 + 32x}$$

E) $2, 1 \pm \sqrt{3}$

$$f(x) = (x-2)((x-(1+\sqrt{3}))((x-(1-\sqrt{3})))$$

$$f(x) = (x-2)((x-1)+\sqrt{3})((x-1)-\sqrt{3})$$

$$f(x) = (x-2)((x-1)^2 - (\sqrt{3})^2)$$

$$f(x) = (x-2)(x^2 - 2x + 1 - 3)$$

$$f(x) = (x-2)(x^2 - 2x - 2)$$

$$\boxed{f(x) = x^3 - 4x^2 + 2x + 4}$$

G) $-2, -1, 0, 1, 2$

$$f(x) = x(x+1)(x-1)(x+2)(x-2)$$

$$f(x) = x(x^2-1)(x^2-4)$$

$$f(x) = x(x^4 - 5x^2 + 4)$$

$$\boxed{f(x) = x^5 - 5x^3 + 4x}$$

H) $1 \pm \sqrt{2}, \pm\sqrt{3}$

$$f(x) = ((x-(1+\sqrt{2}))((x-(1-\sqrt{2})))(x+\sqrt{3})(x-\sqrt{3})$$

$$f(x) = ((x-1)+\sqrt{2})((x-1)-\sqrt{2})(x^2-3)$$

$$f(x) = ((x-1)^2 - (\sqrt{2})^2)(x^2-3)$$

$$f(x) = (x^2 - 2x + 1 - 2)(x^2 - 3)$$

$$f(x) = (x^2 - 2x - 1)(x^2 - 3)$$

$$f(x) = x^4 - 2x^3 - x^2 - 3x^2 + 6x + 3$$

$$\boxed{f(x) = x^4 - 2x^3 - 4x^2 + 6x + 3}$$