Lesson 1.11 - Writing polynomial equations (real zeros)

Learning Objectives: SWBAT
- Write the equation of a polynomial function given its real zeros and multiplicity

Making a connection
- In lesson 1.9 we found all of the real zeros of a polynomial function equation by using factors. From the factors, we also determined a zero's multiplicity
- In this lesson we will be working "backwards" to create function equations by using real zeros and multiplicity to create (and then multiply) factors

Examples: Write the equations of the following functions given the zeros:

a. $-\dfrac{1}{2}, 3, 3$ b. $3, 2 + \sqrt{11}, 2 - \sqrt{11}$

Solution

a. Note that the zero $x = -\frac{1}{2}$ corresponds to either $\left(x + \frac{1}{2}\right)$ or $(2x + 1)$. To avoid fractions, choose the second factor and write

$$f(x) = (2x + 1)(x - 3)^2$$
$$= (2x + 1)(x^2 - 6x + 9) = 2x^3 - 11x^2 + 12x + 9.$$

b. For each of the given zeros, form a corresponding factor and write

$$f(x) = (x - 3)\left[x - \left(2 + \sqrt{11}\right)\right]\left[x - \left(2 - \sqrt{11}\right)\right]$$
$$= (x - 3)\left[(x - 2) - \sqrt{11}\right]\left[(x - 2) + \sqrt{11}\right]$$
$$= (x - 3)\left[(x - 2)^2 - \left(\sqrt{11}\right)^2\right]$$
$$= (x - 3)(x^2 - 4x + 4 - 11)$$
$$= (x - 3)(x^2 - 4x - 7) = x^3 - 7x^2 + 5x + 21.$$

Your Turn: Write the equation of the following functions given the zeros:

$-1, 2 + \sqrt{5}, 2 - \sqrt{5}$

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Practice: Write the equations of the following functions given the zeros:

49. $0, 4$

50. $-7, 2$

51. $0, -2, -3$

52. $0, 2, 5$

53. $4, -3, 3, 0$

54. $-2, -1, 0, 1, 2$

55. $1 + \sqrt{3}, 1 - \sqrt{3}$

56. $6 + \sqrt{3}, 6 - \sqrt{3}$

57. $2, 4 + \sqrt{5}, 4 - \sqrt{5}$

58. $4, 2 + \sqrt{7}, 2 - \sqrt{7}$

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Practice: Write the equations of the following functions given the zeros/multiplicity:

59. Zero: -2, multiplicity: 2
Zero: -1, multiplicity: 1

60. Zero: 3, multiplicity: 1
Zero: 2, multiplicity: 3

61. Zero: -4, multiplicity: 2
Zero: 3, multiplicity: 2

62. Zero: -5, multiplicity: 3
Zero: 0, multiplicity: 2

63. Zero: -1, multiplicity: 2
Zero: -2, multiplicity: 1

64. Zero: -1, multiplicity: 2
Zero: 4, multiplicity: 2

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Practice: Write the equations of the following functions given the zeros/multiplicity:

C) $\pm 1, \pm\sqrt{2}$

F) $\pm 4, 0, \pm\sqrt{2}$

E) $2, 1\pm\sqrt{3}$

G) $-2, -1, 0, 1, 2$

H) $1\pm\sqrt{2}, \pm\sqrt{3}$