Lesson 1.12 - Polynomial Long Division

Learning Objectives: SWBAT

- Divide polynomials using the process of long division and quantify any remainders

Background:

Consider the graph of

$$f(x) = 6x^3 - 19x^2 + 16x - 4.$$

Notice in Figure 2.32 that $x = 2$ appears to be a zero of f. Because $f(2) = 0$, you know that $x = 2$ is a zero of the polynomial function f, and that $(x - 2)$ is a factor of $f(x)$. This means that there exists a second-degree polynomial $q(x)$ such that $f(x) = (x - 2) \cdot q(x)$. To find $q(x)$, you can use **long division of polynomials.**

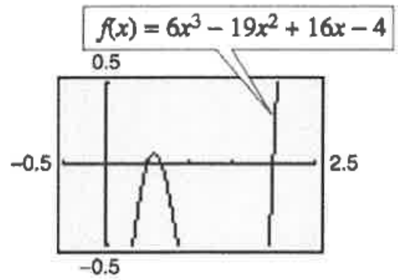

Figure 2.32

Example: Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, and use the result to factor the polynomial completely.

Solution

Partial quotients

$$\begin{array}{rrrrrrrl}
 & & 6x^2 & - & 7x & + & 2 & \\
x - 2 \,\big) & 6x^3 & - 19x^2 & + & 16x & - & 4 & \\
 & \underline{6x^3} & \underline{- 12x^2} & & & & & \text{Multiply: } 6x^2(x - 2). \\
 & & - 7x^2 & + & 16x & & & \text{Subtract.} \\
 & & \underline{- 7x^2} & \underline{+} & \underline{14x} & & & \text{Multiply: } -7x(x - 2). \\
 & & & & 2x & - & 4 & \text{Subtract.} \\
 & & & & \underline{2x} & \underline{-} & \underline{4} & \text{Multiply: } 2(x - 2). \\
 & & & & & & 0 & \text{Subtract.}
\end{array}$$

You can see that

$$\begin{aligned} 6x^3 - 19x^2 + 16x - 4 &= (x - 2)(6x^2 - 7x + 2) \\ &= (x - 2)(2x - 1)(3x - 2). \end{aligned}$$

Note that this factorization agrees with the graph of f (see Figure 2.32) in that the three x-intercepts occur at $x = 2$, $x = \frac{1}{2}$, and $x = \frac{2}{3}$.

Lesson 1.12 - Polynomial Long Division

Use long division to divide the following (please note any remainders)

1. Divide $2x^2 + 10x + 12$ by $x + 3$.

$x+3 \overline{)2x^2+10x+12}$, quotient $2x+4$

$-(2x^2+6x)$

$4x+12$

$-(4x+12)$

0

$\boxed{2x+4}$

2. Divide $5x^2 - 17x - 12$ by $x - 4$.

$x-4 \overline{)5x^2-17x-12}$, quotient $5x+3$

$-(5x^2-20x)$

$3x-12$

$-(3x-12)$

0

$\boxed{5x+3}$

3. Divide $x^4 + 5x^3 + 6x^2 - x - 2$ by $x + 2$.

$x+2 \overline{)x^4+5x^3+6x^2-x-2}$, quotient x^3+3x^2+0x-1

$-(x^4+2x^3)$

$3x^3+6x^2$

$-(3x^3+6x^2)$

$0 - x$

$- 0 \; 0$

$-x-2$

$-(-x-2)$

0

$\boxed{x^3+3x^2-1}$

4. Divide $x^3 - 4x^2 - 17x + 6$ by $x - 3$.

$x-3 \overline{)x^3-4x^2-17x+6}$, quotient x^2-x-20

$-(x^3-3x^2)$

$-x^2-17x$

$(-x^2+3x)$

$-20x+6$

$-(-20x+60)$

-54

Remainder

$\boxed{x^2-x-20-\frac{54}{x-3}}$

5. Divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$.

$4x+5 \overline{)4x^3-7x^2-11x+5}$, quotient x^2-3x+1

$-(4x^3+5x^2)$

$-12x^2-11x$

$-12x^2-15x$

$4x+5$

$-(4x+5)$

0

$\boxed{x^2-3x+1}$

6. Divide $2x^3 - 3x^2 - 50x + 75$ by $2x - 3$.

$2x-3 \overline{)2x^3-3x^2-50x+75}$, quotient $x^2 - 25$

$-(2x^3-3x^2)$

$0 - 50x + 75$

$-(-50x+75)$

0

$\boxed{x^2-25}$

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Use long division to divide the following (please note any remainders)

7. Divide $7x^3 + 3$ by $x + 2$.

$$x+2\,\overline{)\,7x^3+0x^2+0x+3}\quad 7x^2-14x+28$$
$$-(7x^3+14x^2)$$
$$-14x^2+0x$$
$$-(-14x^2-28x)$$
$$28x+3$$
$$-(28x+56)$$
$$\boxed{-53}$$

Remainder

$$\boxed{7x^2-14x+28-\frac{53}{x+2}}$$

8. Divide $8x^4 - 5$ by $2x + 1$.

$$2x+1\,\overline{)\,8x^4+0x^3+0x^2+0x-5}\quad 4x^3-2x^2+x-\tfrac{1}{2}$$
$$-(8x^4+4x^3)$$
$$-4x^3+0x^2$$
$$-(-4x^3-2x^2)$$
$$2x^2+0x$$
$$-(2x^2+x)$$
$$-x-5$$
$$-x-.5$$
$$\boxed{-4.5}$$

Remainder

$$4x^3-2x^2+x-\frac{1}{2}-\frac{4.5}{2x+1}$$

9. $(x + 8 + 6x^3 + 10x^2) \div (2x^2 + 1)$

$$2x^2+0x+1\,\overline{)\,6x^3+10x^2+x+8}\quad 3x+5$$
$$-(6x^3+0x^2+3x)$$
$$10x^2-2x+8$$
$$-(10x^2+0x+5)$$
$$\boxed{-2x+3}$$

remainder

$$\boxed{3x+5-\frac{2x+3}{2x^2+1}}$$

10. $(1 + 3x^2 + x^4) \div (3 - 2x + x^2)$

$$x^2-2x+3\,\overline{)\,x^4+0x^3+3x^2+0x+1}\quad x^2+2x+4$$
$$-(x^4-2x^3+3x^2)$$
$$2x^3+0x^2+0x$$
$$-(2x^3-4x^2+6x)$$
$$4x^2-6x+1$$
$$-(4x^2-8x+12)$$
$$\boxed{2x-11}$$

remainder

~~x^2+2x+4~~

$$\boxed{x^2+2x+4+\frac{2x-11}{x^2-2x+3}}$$

11. $(x^3 - 9) \div (x^2 + 1)$

$$x^2+0x+1\,\overline{)\,x^3+0x^2+0x-9}\quad x$$
$$-(x^3+0x^2+x)$$
$$-x-9$$

$$\boxed{x-\frac{x-9}{x^2+1}}$$

12. $(x^5 + 7) \div (x^3 - 1)$

$$x^3+0x^2+0x-1\,\overline{)\,x^5+0x^4+0x^3+0x^2+0x+7}\quad x^2$$
$$-(x^5+0x^4+0x^3-x^2)$$
$$-x^2+7$$

$$\boxed{x^2-\frac{x^2-7}{x^3-1}}$$