Lesson 1.12 - Polynomial Long Division

Learning Objectives: SWBAT

- Divide polynomials using the process of long division and quantify any remainders

Background:

Consider the graph of

$$f(x) = 6x^3 - 19x^2 + 16x - 4.$$

Notice in Figure 2.32 that $x = 2$ appears to be a zero of f. Because $f(2) = 0$, you know that $x = 2$ is a zero of the polynomial function f, and that $(x - 2)$ is a factor of $f(x)$. This means that there exists a second-degree polynomial $q(x)$ such that $f(x) = (x - 2) \cdot q(x)$. To find $q(x)$, you can use **long division of polynomials.**

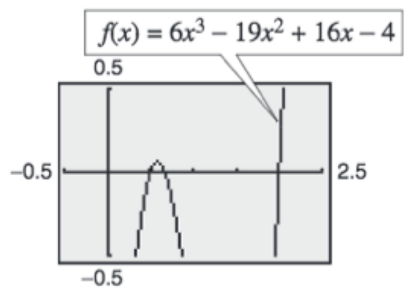

Figure 2.32

Example: Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, and use the result to factor the polynomial completely.

Solution

Partial quotients

$$
\begin{array}{r|l}
 & \phantom{6x^3 - {}} 6x^2 - 7x + 2 \\
x - 2 & \overline{6x^3 - 19x^2 + 16x - 4} \\
 & \underline{6x^3 - 12x^2} \qquad \text{Multiply: } 6x^2(x - 2). \\
 & - 7x^2 + 16x \qquad \text{Subtract.} \\
 & \underline{- 7x^2 + 14x} \qquad \text{Multiply: } -7x(x - 2). \\
 & \phantom{6x^3 - 7x^2 + {}} 2x - 4 \qquad \text{Subtract.} \\
 & \phantom{6x^3 - 7x^2 + {}} \underline{2x - 4} \qquad \text{Multiply: } 2(x - 2). \\
 & \phantom{6x^3 - 7x^2 + 2x - {}} 0 \qquad \text{Subtract.}
\end{array}
$$

You can see that

$$
\begin{aligned}
6x^3 - 19x^2 + 16x - 4 &= (x - 2)(6x^2 - 7x + 2) \\
&= (x - 2)(2x - 1)(3x - 2).
\end{aligned}
$$

Note that this factorization agrees with the graph of f (see Figure 2.32) in that the three x-intercepts occur at $x = 2$, $x = \frac{1}{2}$, and $x = \frac{2}{3}$.

<u>Lesson 1.12 - Polynomial Long Division</u>

Use long division to divide the following (please note any remainders)

1. Divide $2x^2 + 10x + 12$ by $x + 3$.

2. Divide $5x^2 - 17x - 12$ by $x - 4$.

3. Divide $x^4 + 5x^3 + 6x^2 - x - 2$ by $x + 2$.

4. Divide $x^3 - 4x^2 - 17x + 6$ by $x - 3$.

5. Divide $4x^3 - 7x^2 - 11x + 5$ by $4x + 5$.

6. Divide $2x^3 - 3x^2 - 50x + 75$ by $2x - 3$.

Lesson 1.12 - Polynomial Long Division

Use long division to divide the following (please note any remainders)

7. Divide $7x^3 + 3$ by $x + 2$.

8. Divide $8x^4 - 5$ by $2x + 1$.

9. $(x + 8 + 6x^3 + 10x^2) \div (2x^2 + 1)$

10. $(1 + 3x^2 + x^4) \div (3 - 2x + x^2)$

11. $(x^3 - 9) \div (x^2 + 1)$

12. $(x^5 + 7) \div (x^3 - 1)$