# Lesson 1.13 - Synthetic Division

Learning Objectives: SWBAT

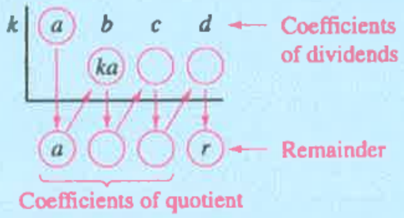
- Divide polynomials using the process of synthetic division

## Background

There is a nice shortcut for long division of polynomials when dividing by divisors of the form $x - k$. The shortcut is called **synthetic division.** The pattern for synthetic division of a cubic polynomial is summarized as follows. (The pattern for higher-degree polynomials is similar.)

**Synthetic Division (of a Cubic Polynomial)**

To divide $ax^3 + bx^2 + cx + d$ by $x - k$, use the following pattern.



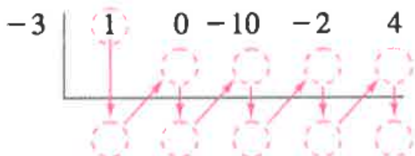

*Vertical pattern:* Add terms.
*Diagonal pattern:* Multiply by $k$.

This algorithm for synthetic division works *only* for divisors of the form $x - k$. Remember that $x + k = x - (-k)$.

**Example** Use synthetic division to divide $x^4 - 10x^2 - 2x + 4$ by $x + 3$.

### Solution

You should set up the array as follows. Note that a zero is included for each missing term in the dividend.




Then, use the synthetic division pattern by adding terms in columns and multiplying the results by $-3$.

Divisor: $x + 3$ Dividend: $x^4 - 10x^2 - 2x + 4$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -10 & -2 & 4 \\ & & -3 & 9 & 3 & -3 \\ \hline & 1 & -3 & -1 & 1 & 1 \end{array}$$

Remainder: 1

Quotient: $x^3 - 3x^2 - x + 1$

So, you have

$$\frac{x^4 - 10x^2 - 2x + 4}{x + 3} = x^3 - 3x^2 - x + 1 + \frac{1}{x + 3}.$$

# Lesson 1.13 - Synthetic Division

Practice: Use synthetic division to divide the polynomial by the given linear factor. Write any remainders as fractions of the divisor

1. $(3x^2 + 7x + 2) \div (x + 2)$

$$\begin{array}{r|rrr} -2 & 3 & 7 & 2 \\ & & -6 & -2 \\ \hline & 3 & 1 & 0 \end{array}$$

$3x+1$

2. $(2x^2 + 7x - 15) \div (x + 5)$

$$\begin{array}{r|rrr} -5 & 2 & 7 & -15 \\ & & -10 & 15 \\ \hline & 2 & -3 & 0 \end{array}$$

$2x-3$

3. $(7x^2 - 3x + 5) \div (x + 1)$

$$\begin{array}{r|rrr} -1 & 7 & -3 & 5 \\ & & -7 & 10 \\ \hline & 7 & -10 & 15 \end{array}$$

$\boxed{7x-10+\frac{15}{x+1}}$

4. $(4x^2 + x + 1) \div (x - 2)$

$$\begin{array}{r|rrr} 2 & 4 & 1 & 1 \\ & & 8 & 18 \\ \hline & 4 & 9 & 19 \end{array}$$

$\boxed{4x+9+\frac{19}{x-2}}$

5. $(3x^2 + 4x - x^4 - 2x^3 - 4) \div (x + 2)$

$-x^4-2x^3+3x^2+4x-4$

$$\begin{array}{r|rrrrr} -2 & -1 & -2 & 3 & 4 & -4 \\ & & 2 & 0 & -6 & 4 \\ \hline & -1 & 0 & 3 & -2 & 0 \end{array}$$

$\boxed{-x^3+3x-2}$

6. $(3x^2 - 4 + x^3) \div (x - 1)$

$x^3+3x^2+0x-4$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

$\boxed{x^2+4x+4}$

7. $(x^4 + 1) \div (x + 1)$

$$\begin{array}{r|rrrrr} -1 & 1 & 0 & 0 & 0 & 1 \\ & & -1 & 1 & -1 & 1 \\ \hline & 1 & -1 & 1 & -1 & 2 \end{array}$$

$\boxed{x^3-x^2+x-1}+\frac{2}{x+1}$

8. $(x^4 + 9) \div (x + 3)$

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & 0 & 9 \\ & & -3 & 9 & -27 & \cancel{-72}\ 81 \\ \hline & 1 & -3 & 9 & -27 & 90 \end{array}$$

$\boxed{x^3-3x^2+9x-27+\frac{90}{x+3}}$

9. $(x^4 - 16) \div (x + 2)$

$$\begin{array}{r|rrrrr} -2 & 1 & 0 & 0 & 0 & -16 \\ & & -2 & 4 & -8 & 16 \\ \hline & 1 & -2 & 4 & -8 & 0 \end{array}$$

$\boxed{x^3-2x^2+4x-8}$

10. $\dfrac{x^6 + 4x^5 - 2x^3 + 7}{x + 1}$

$$\begin{array}{r|rrrrrrr} -1 & 1 & 4 & 0 & -2 & 0 & 0 & 7 \\ & & -1 & -3 & 3 & -1 & 1 & -1 \\ \hline & 1 & 3 & -3 & 1 & -1 & 1 & 6 \end{array}$$

$\boxed{x^5+3x^4-3x^3+x^2-x+1}+\frac{6}{x+1}$