Learning Objectives: SWBAT

- · Use the remainder theorem to evaluate a polynomial for a given input value
- Use the factor theorem to determine if a linear binomial is a factor of a larger polynomial
- · Determine all real zeros of a polynomial given a factor

Background - The Remainder Theorem

The remainder obtained in the synthetic division process has an important interpretation, as described in the **Remainder Theorem.**

The Remainder Theorem (See the proof on page 180.) If a polynomial f(x) is divided by x - k, the remainder is r = f(k).

The Remainder Theorem tells you that synthetic division can be used to evaluate a polynomial function. That is, to evaluate a polynomial function f(x) when x = k, divide f(x) by x - k. The remainder will be f(k).

Example: Use the Remainder Theorem to evaluate the following function at x = -2.

 $f(x) = 3x^3 + 8x^2 + 5x - 7$

Solution

Using synthetic division, you obtain the following.

-2	3	8	5	-7
		-6	-4	-2
	3	2	1	-9

Because the remainder is r = -9, you can conclude that

 $f(-2) = -9. \qquad r = f(k)$

This means that (-2, -9) is a point on the graph of f. You can check this by substituting x = -2 in the original function.

Check

$$f(-2) = 3(-2)^3 + 8(-2)^2 + 5(-2) - 7$$

= 3(-8) + 8(4) - 10 - 7
= -24 + 32 - 10 - 7 = -9

Background - The Factor Theorem

Another important theorem is the **Factor Theorem.** This theorem states that you can test whether a polynomial has (x - k) as a factor by evaluating the polynomial at x = k. If the result is 0, (x - k) is a factor.

A polynomial f(x) has a factor (x - k) if and only if f(k) = 0.

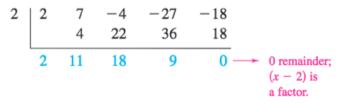
Example: Show that (x - 2) and (x + 3) are factors of

 $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18.$

Then find the remaining factors of f(x).

Algebraic Solution

Using synthetic division with the factor (x - 2), you obtain the following.



Take the result of this division and perform synthetic division again using the factor (x + 3).

Because the resulting quadratic factors as

 $2x^2 + 5x + 3 = (2x + 3)(x + 1)$

the complete factorization of f(x) is

f(x) = (x - 2)(x + 3)(2x + 3)(x + 1).

Using the Remainder in Synthetic Division

In summary, the remainder r, obtained in the synthetic division of f(x) by x - k, provides the following information.

1. The remainder r gives the value of f at x = k. That is, r = f(k).

2. If r = 0, (x - k) is a factor of f(x).

3. If r = 0, (k, 0) is an x-intercept of the graph of f.

<u>Practice</u> In Exercises 35–38, use the Remainder Theorem and synthetic division to evaluate the function at each given value. Use a graphing utility to verify your results.

35. $f(x) = 2x^3 - 7x + 3$ (a) f(1) (b) f(-2) (c) $f(\frac{1}{2})$ (d) f(2)

36.
$$g(x) = 2x^6 + 3x^4 - x^2 + 3$$

(a) $g(2)$ (b) $g(1)$ (c) $g(3)$ (d) $g(-1)$

37.
$$h(x) = x^3 - 5x^2 - 7x + 4$$

(a) $h(3)$ (b) $h(2)$ (c) $h(-2)$ (d) $h(-5)$

<u>Practice</u> In Exercises 39-42, use synthetic division to show that x is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all the real zeros of the function.

Polynomial EquationValue of x**39.** $x^3 - 7x + 6 = 0$ x = 2

40. $x^3 - 28x - 48 = 0$ x = -4

41. $2x^3 - 15x^2 + 27x - 10 = 0$ $x = \frac{1}{2}$

State if the given binomial is a factor of the given polynomial.

7)
$$(k^3 - k^2 - k - 2) \div (k - 2)$$

8) $(b^4 - 8b^3 - b^2 + 62b - 34) \div (b - 7)$

9)
$$(n^4 + 9n^3 + 14n^2 + 50n + 9) \div (n + 8)$$

10) $(p^4 + 6p^3 + 11p^2 + 29p - 13) \div (p + 5)$

Practice

Factor each and find all zeros. One factor has been given.

1)
$$f(x) = x^3 + 9x^2 + 23x + 15$$
; $x + 5$
2) $f(x) = x^3 - x^2 - 14x + 24$; $x - 3$

3)
$$f(x) = x^4 + 3x^3 - 13x^2 - 15x$$
; $x - 3$
4) $f(x) = x^3 - 12x^2 + 47x - 60$; $x - 3$

5)
$$f(x) = x^3 - 7x^2 + 2x + 40; x - 5$$

6) $f(x) = x^3 - 3x^2 - 9x + 27; x - 3$

9)
$$f(x) = 5x^3 + 21x^2 - 21x - 5$$
; $x + 5$
10) $f(x) = 3x^3 - 4x^2 - 9x + 10$; $x - 2$