Lesson 1.18 - The Fundamental Theorem of Algebra (part 2)

Learning Objectives: SWBAT:

1. Use the Fundamental Theorem of Algebra to write the equation of a polynomial given real and/or complex zeros

Making a connection:

- In lesson 1.11 we learned how to write the equation of a polynomial given its REAL zeros.
- This lesson is similar, except now we will also be given COMPLEX zeros.
- One of the keys to this is remembering that complex zeros (and irrational zeros), ALWAYS come in conjugate pairs.
- Therefore, if we are given one complex or irrational zero, we need to remember that its conjugate is also a zero

Example: Find a *fourth-degree* polynomial function with real coefficients that has -1, -1, and $3i$ as zeros.

Solution

Because $3i$ is a zero *and* the polynomial is stated to have real coefficients, you know that the conjugate $-3i$ must also be a zero. So, from the Linear Factorization Theorem, $f(x)$ can be written as

$$f(x) = a(x+1)(x+1)(x-3i)(x+3i).$$

For simplicity, let $a = 1$ to obtain

$$f(x) = (x^2+2x+1)(x^2+9) = x^4 + 2x^3 + 10x^2 + 18x + 9.$$

Practice: Write the equation of the polynomial given the zeros:

37. $2, i, -i$

$(x-2)(x+i)(x-i)$

$(x-2)(x^2+1)$

$\boxed{f(x) = x^3 - 2x^2 + x - 2}$

38. $3, 4i, -4i$

$(x-3)(x+4i)(x-4i)$

$(x-3)(x^2+16)$

$\boxed{f(x) = x^3 - 3x^2 + 16x - 48}$

39. $2, 2, 4 - i$

$(x-2)(x-2)(x-(4-i))(x-4+i)$

$(x^2-4x+4)(x^2-8x+17)$

$\boxed{f(x) = x^4 - 12x^3 + 53x^2 - 100x + 68}$

40. $-1, -1, 2 + 5i$

$(x+1)(x+1)(x-(2+5i))(x-(2-5i))$

$(x^2+2x+1)(x^2-4x+29)$

$\boxed{f(x) = x^4 - 2x^3 + 22x^2 + 54x + 29}$

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Practice: Write the equation of the polynomial given the zeros:

41. $0, -5, 1+\sqrt{2}i$

$x(x+5)(x-(1+i\sqrt{2}))(x-(1-i\sqrt{2}))$

$(x^2+5x)(x^2-2x+3)$

$\boxed{f(x) = x^4+3x^3-7x^2+15x}$

42. $0, 4, 1+\sqrt{2}i$

$x(x-4)(x-1+i\sqrt{2})(x-1-i\sqrt{2})$

$(x^2-4x)(x^2-2x+3)$

$\boxed{f(x) = x^4-6x^3+11x^2-12x}$

43. $1, -2, 2i$

$(x-1)(x+2)(x+2i)(x-2i)$

$(x^2+x-2)(x^2+4)$

$\boxed{f(x) = x^4+x^3+2x^2+4x-8}$

44. $-1, 2, i$

$(x+1)(x-2)(x+i)(x-i)$

$(x^2-x-2)(x^2+1)$

$\boxed{f(x) = x^4-x^3-x^2-x-2}$

45. $-1, 2+\sqrt{5}i$

$(x+1)(x-(2+i\sqrt{5}))(x-(2-i\sqrt{5}))$

$(x-1)(x^2-4x+9)$

$\boxed{f(x) = x^3-5x^2+13x-9}$

46. $-2, 2+2\sqrt{2}i$

$(x+2)(x-(2+i\sqrt{2}))(x-2-i\sqrt{2})$

$(x+2)(x^2-4x+12)$

$\boxed{f(x) = x^3-2x^2+4x+24}$

State the **possible** number of imaginary zeros of $g(x) = x^5+ax^4+bx^3+cx^2+dx+e$, where *a, b, c, d,* and *e* are real number coefficients.

(A) 3 or 1 (B) 2, 4, or 0 (C) Exactly 1 (D) Exactly 3 (E) Exactly 4

there always needs to be an even # of complex solutions b/c they always come in conjugate pairs