Lesson 1.18 - The Fundamental Theorem of Algebra (part 2)

Learning Objectives: SWBAT:

1. Use the Fundamental Theorem of Algebra to write the equation of a polynomial given real and/or complex zeros

Making a connection:

- In lesson 1.11 we learned how to write the equation of a polynomial given its REAL zeros.
- This lesson is similar, except now we will also be given COMPLEX zeros.
- One of the keys to this is remembering that complex zeros (and irrational zeros), ALWAYS come in conjugate pairs.
- Therefore, if we are given one complex or irrational zero, we need to remember that its conjugate is also a zero

Example: Find a *fourth-degree* polynomial function with real coefficients that has -1, -1, and $3i$ as zeros.

Solution

Because $3i$ is a zero *and* the polynomial is stated to have real coefficients, you know that the conjugate $-3i$ must also be a zero. So, from the Linear Factorization Theorem, $f(x)$ can be written as

$$f(x) = a(x + 1)(x + 1)(x - 3i)(x + 3i).$$

For simplicity, let $a = 1$ to obtain

$$f(x) = (x^2 + 2x + 1)(x^2 + 9) = x^4 + 2x^3 + 10x^2 + 18x + 9.$$

Practice: Write the equation of the polynomial given the zeros:

37. $2, i, -i$

38. $3, 4i, -4i$

39. $2, 2, 4 - i$

40. $-1, -1, 2 + 5i$

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Practice: Write the equation of the polynomial given the zeros:

41. $0, -5, 1 + \sqrt{2}i$

42. $0, 4, 1 + \sqrt{2}i$

43. $1, -2, 2i$

44. $-1, 2, i$

45. $-1, 2 + \sqrt{5}i$

46. $-2, 2 + 2\sqrt{2}i$

State the **possible** number of imaginary zeros of $g(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$, where *a, b, c, d,* and *e* are real number coefficients.

(A) 3 or 1 (B) 2, 4, or 0 (C) Exactly 1 (D) Exactly 3 (E) Exactly 4