Lesson 1.19 - The Fundamental Theorem of Algebra (part 3)

Learning Objectives: SWBAT:

1. Use the Fundamental Theorem of Algebra to determine all zeros given complex or irrational zeros

Making a connection:

- In this lesson, we will be given either a complex or irrational zero of a polynomial equation
- We need to remember that irrational and complex zeros ALWAYS come in conjugate pairs
- We will multiply these pairs together, then use long division to break down the polynomial into its remaining zeros

Example: Find all the zeros of

$$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$$

given that $1 + 3i$ is a zero of f.

Algebraic Solution

Because complex zeros occur in conjugate pairs, you know that $1 - 3i$ is also a zero of f. This means that both

$$x - (1 + 3i) \qquad \text{and} \qquad x - (1 - 3i)$$

are factors of f. Multiplying these two factors produces

$$\begin{aligned}[x - (1 + 3i)][x - (1 - 3i)] &= [(x - 1) - 3i][(x - 1) + 3i] \\ &= (x - 1)^2 - 9i^2 \\ &= x^2 - 2x + 10.\end{aligned}$$

Using long division, you can divide $x^2 - 2x + 10$ into f to obtain the following.

$$\begin{array}{r|l} & x^2 - x - 6 \\ x^2 - 2x + 10 & x^4 - 3x^3 + 6x^2 + 2x - 60 \\ & \underline{x^4 - 2x^3 + 10x^2} \\ & -x^3 - 4x^2 + 2x \\ & \underline{-x^3 + 2x^2 - 10x} \\ & -6x^2 + 12x - 60 \\ & \underline{-6x^2 + 12x - 60} \end{array}$$

So, you have

$$\begin{aligned}f(x) &= (x^2 - 2x + 10)(x^2 - x - 6) \\ &= (x^2 - 2x + 10)(x - 3)(x + 2)\end{aligned}$$

and you can conclude that the zeros of f are $x = 1 + 3i$, $x = 1 - 3i$, $x = 3$, and $x = -2$.

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Practice: Given one zero, determine all remaining zeros of the polynomial

	Function	*Zero*
51.	$f(x) = 2x^3 + 3x^2 + 50x + 75$	$5i$
52.	$f(x) = x^3 + x^2 + 9x + 9$	$3i$
53.	$g(x) = x^3 - 7x^2 - x + 87$	$5 + 2i$

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Practice: Given one zero, determine all remaining zeros of the polynomial

	Function	*Zero*
54.	$g(x) = 4x^3 + 23x^2 + 34x - 10$	$-3 + i$

55. $h(x) = 3x^3 - 4x^2 + 8x + 8$ $\quad$ $1 - \sqrt{3}i$

56. $f(x) = x^3 + 4x^2 + 14x + 20$ $\quad$ $-1 - 3i$

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Practice: Given one zero, determine all remaining zeros of the polynomial

	Function	*Zero*
59.	$f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22$	$-3 + i\sqrt{2}$

60. f(x) = $x^4 + 2x^3 + x^2 + 18x - 72$ 3i