Lesson 1.1 - Review of Functions "Basics"

Learning Objectives: SWBAT

- Explain how/why the relationship between two variables is classified as a "function"
- Determine whether or not a set of ordered pairs is a function
- Test for functions Algebraically
- Evaluate a function for a given input value

Introduction to Functions

Many everyday phenomena involve pairs of quantities that are related to each other by some rule of correspondence. The mathematical term for such a rule of correspondence is a **relation.** Here are two examples.

1. The simple interest I earned on an investment of \$1000 for 1 year is related to the annual interest rate r by the formula $I = 1000r$.
2. The area A of a circle is related to its radius r by the formula $A = \pi r^2$.

Not all relations have simple mathematical formulas. For instance, people commonly match up NFL starting quarterbacks with touchdown passes, and hours of the day with temperature. In each of these cases, there is some relation that matches each item from one set with exactly one item from a different set. Such a relation is called a **function.**

> **Definition of a Function**
>
> A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

To help understand this definition, look at the function that relates the time of day to the temperature in Figure 1.14.

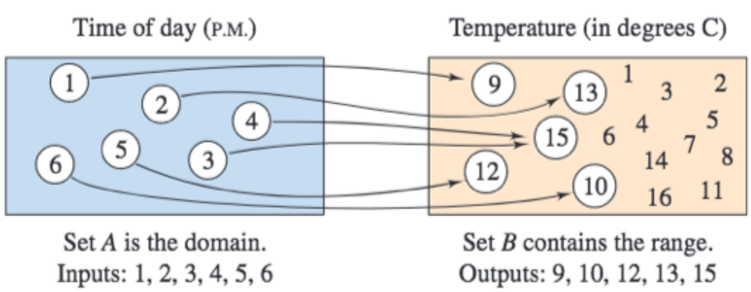


Figure 1.14

This function can be represented by the ordered pairs $\{(1, 9°), (2, 13°), (3, 15°), (4, 15°), (5, 12°), (6, 10°)\}$. In each ordered pair, the first coordinate (x-value) is the **input** and the second coordinate (y-value) is the **output.**

> **Characteristics of a Function from Set A to Set B**
>
> 1. Each element of A must be matched with an element of B.
> 2. Some elements of B may not be matched with any element of A.
> 3. Two or more elements of A may be matched with the same element of B.
> 4. An element of A (the domain) cannot be matched with two different elements of B.

Lesson 1.1 - Review of Functions "Basics"

<u>The 4 ways to define functions</u>

1. *Verbally* by a sentence that describes how the input variables are related to the output variables

   Example: The input value x is the election year from 1952 to 2004 and the output value y is the elected president of the United States.
2. *Numerically* by a table or a list of ordered pairs that matches input values with output values

   Example: In the set of ordered pairs $\{(2, 34), (4, 40), (6, 45), (8, 50), (10, 54)\}$, the input value is the age of a male child in years and the output value is the height of the child in inches.
3. *Graphically* by points on a graph in a coordinate plane in which the input values are represented by the horizontal axis and the output values are represented by the vertical axis

   Example: See Figure 1.15.
4. *Algebraically* by an equation in two variables

   Example: The formula for temperature, $F = \frac{9}{5}C + 32$, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius, is an equation that represents a function. You will see that it is often convenient to approximate data using a mathematical model or formula.

<u>Testing for functions</u> - Example 1 (Ordered Pairs)

Decide whether the relation represents y as a function of x.

a.

Input, x	2	2	3	4	5
Output, y	11	10	8	5	1

b.

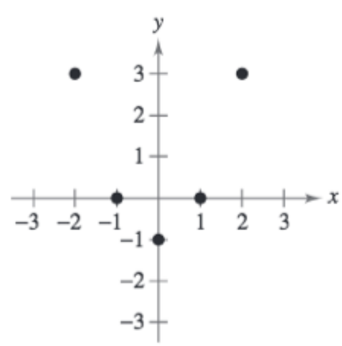


Figure 1.15

Solution

a. This table *does not* describe y as a function of x. The input value 2 is matched with two different y-values.

b. The graph in Figure 1.15 *does* describe y as a function of x. Each input value is matched with exactly one output value.

Lesson 1.1 - Review of Functions "Basics"

Testing for functions - Example 2 (Algebraically)

Which of the equations represent(s) y as a function of x?

a. $x^2 + y = 1$ **b.** $-x + y^2 = 1$

Solution

To determine whether y is a function of x, try to solve for y in terms of x.

a. Solving for y yields

$$x^2 + y = 1 \quad \text{Write original equation.}$$

$$y = 1 - x^2. \quad \text{Solve for } y.$$

Each value of x corresponds to exactly one value of y. So, y is a function of x.

b. Solving for y yields

$$-x + y^2 = 1 \quad \text{Write original equation.}$$

$$y^2 = 1 + x \quad \text{Add } x \text{ to each side.}$$

$$y = \pm\sqrt{1 + x}. \quad \text{Solve for } y.$$

The $\pm$ indicates that for a given value of x there correspond two values of y. For instance, when $x = 3$, $y = 2$ or $y = -2$. So, y is not a function of x.

Evaluating Functions (just plug in the input value)

Let $g(x) = -x^2 + 4x + 1$. Find (a) $g(2)$, (b) $g(t)$, and (c) $g(x + 2)$.

Solution

a. Replacing x with 2 in $g(x) = -x^2 + 4x + 1$ yields the following.

$$g(2) = -(2)^2 + 4(2) + 1 = -4 + 8 + 1 = 5$$

b. Replacing x with t yields the following.

$$g(t) = -(t)^2 + 4(t) + 1 = -t^2 + 4t + 1$$

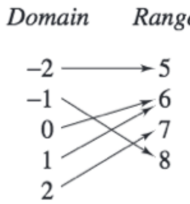
c. Replacing x with $x + 2$ yields the following.

$$\begin{aligned} g(x + 2) &= -(x + 2)^2 + 4(x + 2) + 1 && \text{Substitute } x + 2 \text{ for } x. \\ &= -(x^2 + 4x + 4) + 4x + 8 + 1 && \text{Multiply.} \\ &= -x^2 - 4x - 4 + 4x + 8 + 1 && \text{Distributive Property} \\ &= -x^2 + 5 && \text{Simplify.} \end{aligned}$$

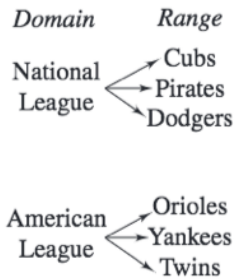
Lesson 1.1 - Review of Functions "Basics"

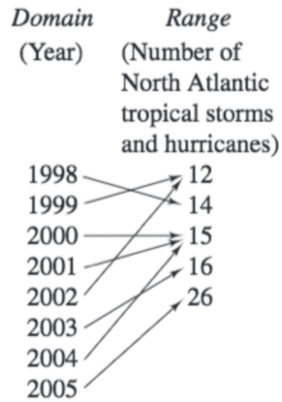
Practice

In Exercises 1–4, does the relation describe a function? Explain your reasoning.

1. 

2. 

3. 

4. 

In Exercises 5–8, decide whether the relation represents y as a function of x. Explain your reasoning.

5.

Input, x	-3	-1	0	1	3
Output, y	-9	-1	0	1	9

6.

Input, x	0	1	2	1	0
Output, y	-4	-2	0	2	4

7.

Input, x	10	7	4	7	10
Output, y	3	6	9	12	15

8.

Input, x	0	3	9	12	15
Output, y	3	3	3	3	3

In Exercises 9 and 10, which sets of ordered pairs represent functions from A to B? Explain.

9. $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1, 2\}$

(a) $\{(0, 1), (1, -2), (2, 0), (3, 2)\}$

(b) $\{(0, -1), (2, 2), (1, -2), (3, 0), (1, 1)\}$

(c) $\{(0, 0), (1, 0), (2, 0), (3, 0)\}$

(d) $\{(0, 2), (3, 0), (1, 1)\}$

10. $A = \{a, b, c\}$ and $B = \{0, 1, 2, 3\}$

(a) $\{(a, 1), (c, 2), (c, 3), (b, 3)\}$

(b) $\{(a, 1), (b, 2), (c, 3)\}$

(c) $\{(1, a), (0, a), (2, c), (3, b)\}$

(d) $\{(c, 0), (b, 0), (a, 3)\}$

Lesson 1.1 - Review of Functions "Basics"

Practice

In Exercises 13–24, determine whether the equation represents y as a function of x.

13. $x^2 + y^2 = 4$

14. $x = y^2 + 1$

15. $y = \sqrt{x^2 - 1}$

16. $y = \sqrt{x + 5}$

17. $2x + 3y = 4$

18. $x = -y + 5$

19. $y^2 = x^2 - 1$

20. $x + y^2 = 3$

In Exercises 27–42, evaluate the function at each specified value of the independent variable and simplify.

27. $f(t) = 3t + 1$

(a) $f(2)$ (b) $f(-4)$ (c) $f(t + 2)$

29. $h(t) = t^2 - 2t$

(a) $h(2)$ (b) $h(1.5)$ (c) $h(x + 2)$

31. $f(y) = 3 - \sqrt{y}$

(a) $f(4)$ (b) $f(0.25)$ (c) $f(4x^2)$

Lesson 1.1 - Review of Functions "Basics"

Practice **In Exercises 27–42, evaluate the function at each specified value of the independent variable and simplify.**

32. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8)$ (b) $f(1)$ (c) $f(x - 8)$

33. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0)$ (b) $q(3)$ (c) $q(y + 3)$

34. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2)$ (b) $q(0)$ (c) $q(-x)$

35. $f(x) = \dfrac{|x|}{x}$

(a) $f(3)$ (b) $f(-3)$ (c) $f(t)$

36. $f(x) = |x| + 4$

(a) $f(4)$ (b) $f(-4)$ (c) $f(t)$