Lesson 1.20 - Domain and Excluded Values of Rational Expressions

Learning Objectives: SWBAT

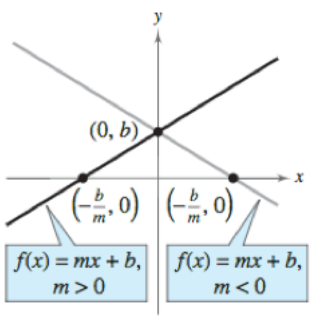
1. Determine whether the graph of an expression is continuous or discontinuous given its equation
2. Determine the domain/excluded values of a rational expression

Making a Connection

- For each of the functions we have studied so far, the graphs have been CONTINUOUS.
- As you may remember, for a graph that is continuous, the domain is "all real numbers" since every possible x value is accounted for on the graph.
- Below is a snapshot of the three parent function graphs we are familiar with:

Linear Function

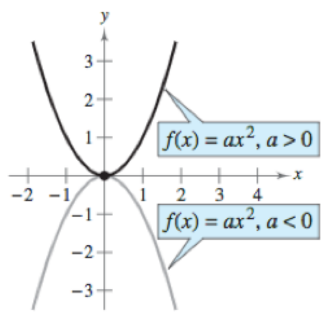
$f(x) = mx + b$



Domain: $(-\infty, \infty)$

Quadratic (Squaring) Function

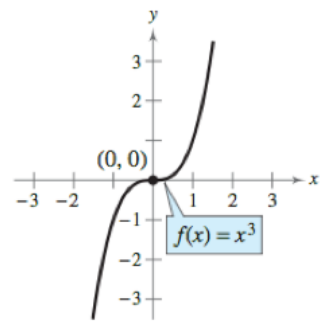
$f(x) = ax^2$



Domain: $(-\infty, \infty)$

Cubic Function

$f(x) = x^3$

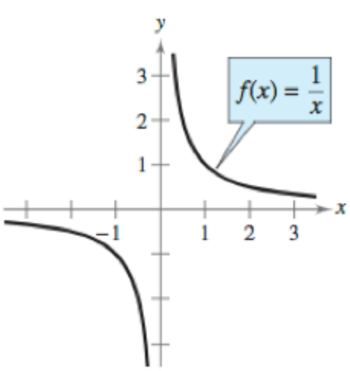


Domain: $(-\infty, \infty)$

- Notice: Each graph has no "gaps" all x values on the number line are accounted for on the graph.
- Rational function graphs are different than the other graphs we have studied because they are NOT continuous, they are discontinuous.
- Below is the parent function equation of a rational function with its graph

Rational (Reciprocal) Function

$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$

Notice: This graph has no x value at 0

> The reason why there is no x value at 0 is because the parent function has a variable in the denominator

> If we substituted a value of zero into the function, we would get something that is undefined (YOU CANT DIVIDE BY ZERO!)

> Therefore, the domain of this function is "All real numbers except for zero".

> x = 0 is called an "excluded value" because it is excluded from the domain

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How to determine the domain and excluded values of a rational expression

- The most important thing is to look to the DENOMINATOR of the fraction. You can ignore the numerator. Remember, the reason that there are excluded values in the first place is because there is a value of x that results in dividing by zero.
- Whatever expressions are found in the denominator, set them equal to zero and solve for the x value. Keep in mind, that many problems will require you to FACTOR FIRST so that you can determine these values. Here are examples:

EXAMPLE Determine the value or values of the variable for which the rational expression is defined.

a) $\frac{x+3}{2x-5}$ b) $\frac{x+3}{x^2+6x-7}$

a) Determine the value or values of x that make $2x - 5$ equal to 0 and exclude these. This can be done by setting $2x - 5$ equal to 0 and solving the equation for x.

$$2x - 5 = 0$$

$$2x = 5 \quad \boxed{x \neq \tfrac{5}{2}.}$$

b) To determine the value or values that are excluded, set the denominator equal to zero and solve the equation for the variable.

$$x^2 + 6x - 7 = 0$$ Factor the denominator!

$$(x + 7)(x - 1) = 0$$

$$x + 7 = 0 \quad \text{or} \quad x - 1 = 0$$

$$\boxed{x \neq -7 \text{ and } x \neq 1}$$

Practice: Determine the excluded values for each expression and state the domain

1) $\frac{60x^3}{12x}$

2) $\frac{70v^2}{100v}$

3) $\frac{m+7}{m^2+4m-21}$

4) $\frac{n^2+6n+5}{n+1}$

5) $\frac{35x-35}{25x-40}$

6) $\frac{-n^2+16n-63}{n^2-2n-35}$

7) $\frac{p+4}{p^2+6p+8}$

8) $\frac{9}{15a-15}$

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Practice: Determine the excluded values for each expression and state the domain

11) $\dfrac{x^2 + x - 6}{x^2 + 8x + 15}$

12) $\dfrac{a^2 + 5a + 4}{a^2 + 9a + 20}$

13) $\dfrac{x^2 - 2x - 15}{x^2 - 6x + 5}$

14) $\dfrac{10x - 6}{10x - 6}$

15) $\dfrac{(v-7)(v+8)}{(v+8)(v-10)} \div \dfrac{1}{v-10}$

16) $\dfrac{n+3}{n+2} \div \dfrac{(n-1)(n+3)}{(n-1)^2}$

17) $\dfrac{x+3}{4} \cdot \dfrac{3(x-6)}{3(x+3)}$

18) $\dfrac{x-8}{(x+6)(x-8)} \cdot \dfrac{4x(x+10)}{x+10}$

19) $\dfrac{2b^2 - 12b}{b+5} \div \dfrac{b-6}{b+5}$

20) $\dfrac{1}{n+9} \div \dfrac{6-n}{3n-18}$

21) $\dfrac{28 - 7b}{b-4} \cdot \dfrac{1}{b+10}$

22) $\dfrac{2}{v^2 - 12v + 27} \cdot \dfrac{v^2 - 12v + 27}{3}$

23) $\dfrac{1}{5p^2} \div \dfrac{9p - 36}{5p^3 - 35p^2}$

24) $\dfrac{8 - 7x - x^2}{x+8} \cdot \dfrac{x+5}{9x-9}$