Lesson 1.21 - Rational Function Attributes

Learning Objectives: SWBAT

1. Identify discontinuities in Rational function graphs as Vertical Asymptotes or holes and determine their location given the function's equation
2. Determine the horizontal Asymptotes of a rational function
3. Determine the zeros of a rational function

What are *Discontinuities*?

- Discontinuities are "gaps" in what would otherwise be a continuous graph. The two types of discontinuities are Vertical Asymptotes and Holes.

What is an *Asymptote*?

- An Asymptote represents the *end behavior* of a curved graph as it approaches either +/- infinity. The graph approaches the asymptote but NEVER touches/crosses it. Asymptotes can vertical, horizontal or oblique (slanted)

What is a *Vertical Asymptote*?

- A Vertical Asymptote is a vertical **line of discontinuity** on the graph. It represents a x values where the y value is undefined.

What is a *hole*?

- A Hole is a **point of discontinuity** on the graph. Similar to vertical asymptotes above, holes represent x value(s) where the y value is undefined.

How do we find Vertical Asymptotes and Holes?

- A Vertical Asymptote is the result when the factor that causes the denominator to equal zero is NOT cancelled while simplifying
- A Hole is the result when the factor that causes the denominator to equal 0 also exists in the numerator (and thereby can be cancelled out)

Example: Identify the number of discontinuities present in the rational function below. Identify the location of any vertical Asymptotes and the coordinates of any holes

$$f(x)=\frac{x-4}{x^2-16} \quad \text{factor the denominator} \longrightarrow \frac{\cancel{(x-4)}}{(x+4)\cancel{(x-4)}} \quad \text{notice how } x = 4 \text{ cancels}$$

- There is a vertical Asymptote at x = -4
- There is a hole at x = 4. This is the x coordinate of the hole

To find the y coordinate of the hole:

> Simplify (cancel) the expression above and plug in the x coordinate of the hole

$$\frac{\cancel{(x-4)}}{(x+4)\cancel{(x-4)}} \longrightarrow \frac{1}{(x+4)} \longrightarrow \frac{1}{((4)+4)} \longrightarrow \frac{1}{8}$$

> The coordinate of the hole is $\left(4, \frac{1}{8}\right)$

Your Turn Determine the locations of any vertical asymptotes and the corrdinates of any hole of the following function

$$f(x) = \frac{x^2 - 9}{x^2 + 4x + 3}$$

$$\frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}(x+1)}$$

y coordinate of hole

$$\frac{(-3)-3}{(-3)+1} = \frac{-6}{-2} = 3$$

VA: x = -1

hole: (-3, 3)

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How do we find Horizontal Asymptotes?

- The existence and location of a horizontal asymptote depends on the one of the following conditions:
  - > If the degree of the numerator is less than the degree of the denominator, then a a horizontal asymptote exists at y = 0
  - > If the degree of the numerator is greater than the degree of the denominator, then a horizontal asymptote does not exist
  - > If the degree of the numerator and denominator are the same then a horizontal asymptote will exist at the following line:

$$y = \frac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$$

Example **Let $y = \frac{3 - 2x - x^2}{x^2 + x - 2}$. Identify all asymptotes and holes in the graph.**

SOLUTION

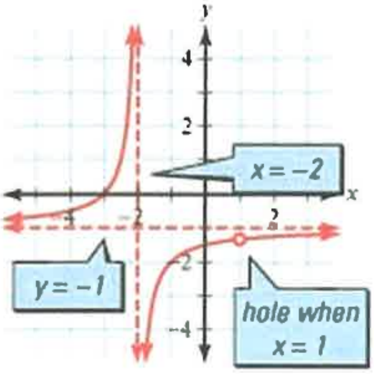
Factor the numerator and denominator.

$$y = \frac{3 - 2x - x^2}{x^2 + x - 2} = \frac{-(x^2 + 2x - 3)}{(x - 1)(x + 2)} = \frac{-(x + 3)\cancel{(x - 1)}}{\cancel{(x - 1)}(x + 2)}$$

Because $x - 1$ is a factor of *both* the numerator and the denominator, the graph has a hole when $x = 1$.

Because $x + 2$ is a factor of *only* the denominator, there is a vertical asymptote at $x = -2$.

Because the degree of the numerator equals the degree of the denominator, there is a horizontal asymptote at $y = \frac{-1}{1} = -1$.



After canceling

How do we find zeros (x intercepts)?

- Look to the numerator. Whatever x value makes the numerator zero is the x value of the x intercept
- In the example above, the coordinates of the x intercept is (-3, 0)

Your Turn - Find any vertical asymptotes, holes, horizontal asymptotes and zeros for the following function:

$$f(x) = \frac{x^3 + 3x^2}{x^2 + 2x - 3} = \frac{x^2\cancel{(x+3)}}{\cancel{(x+3)}(x-2)}$$

Ycoordinate of hole

$$\frac{(-3)^2}{(-3)-2} = \frac{9}{-5}$$

VA: $x = 2$

Hole: $(-3, \frac{9}{5})$

HA: none

Zeros: $x = 0$ mult 2

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For each function: Find any vertical asymptotes, holes, horizontal asymptotes and zeros for the following function:

1) $f(x) = \dfrac{1}{3x^2+3x-18} = \dfrac{1}{3(x^2+x-6)} = \dfrac{1}{3(x+3)(x-2)}$

VA: $x = -3, 2$

holes: none

HA: $y = 0$

zeros: none

2) $f(x) = \dfrac{x-2}{x-4}$

VA: $x = 4$

holes: none

HA: $y = 1$

zeros: $x = 2$

3) $f(x) = \dfrac{x^3-x^2-6x}{-3x^2-3x+18}$ $\dfrac{x(x-3)(x+2)}{-3(x+3)(x-2)}$

VA: $x = 2, -3$

holes: none

HA: none

zeros: $x = 0, -2, 3$

4) $f(x) = \dfrac{x^2+x-6}{-4x^2-16x-12}$ $\dfrac{\cancel{(x+3)}(x-2)}{-4\cancel{(x+3)}(x+1)}$

VA: $x = -1$

hole: $(-3, \frac{-5}{8})$

HA: $y = \frac{-1}{4}$

zeros: $x = 2$

y coordinate

$\dfrac{(-3)-2}{-4(-3+1)} = \dfrac{-5}{8}$

5) $f(x) = -\dfrac{4}{x^2-3x}$ $\dfrac{-4}{x(x-3)}$

VA: $x = 0, 3$

holes: none

HA: $y = 0$

zeros: none

6) $f(x) = \dfrac{x-4}{-4x-16} = \dfrac{x-4}{-4(x+4)}$

VA: $x = -4$

holes: none

HA $y = \frac{-1}{4}$

zeros: $x = 4$

7) $f(x) = \dfrac{x+4}{-2x-6}$ $\dfrac{x+4}{-2(x+3)}$

VA: $x = -3$

holes: none

HA: $y = \frac{-1}{2}$

zeros: $x = -4$

8) $f(x) = \dfrac{x^3-9x}{3x^2-6x-9}$ $\dfrac{x(x+3)\cancel{(x-3)}}{3(x+1)\cancel{(x-3)}}$

VA: $x = -1$

hole: $(3, \frac{3}{2})$

HA: none

zeros: $x = 0, -3$

Y coordinate

$\dfrac{3(3+3)}{3(3+1)} = \dfrac{18}{12} = \dfrac{3}{2}$

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9) $f(x) = \dfrac{3x^2 - 12x}{x^2 - 2x - 3} = \dfrac{3x(x-4)}{(x-3)(x+1)}$

VA: $x = 3, -1$

holes: none

HA: $y = 3$

Zeros: $x = 0, 4$

10) $f(x) = \dfrac{x^3 - 16x}{-4x^2 + 4x + 24}$ $\dfrac{x(x+4)(x-4)}{-4(x-3)(x+2)}$

VA: $x = 3, -2$

holes: none

HA: none

Zeros: $x = 0, 4, -4$

11) $f(x) = \dfrac{x^2 + 2x}{-4x + 8}$ $\dfrac{x(x+2)}{-4(x-2)}$

VA: $x = 2$

holes: none

HA: none

Zeros: $x = 0, -2$

12) $f(x) = \dfrac{x + 2}{2x + 6}$ $\dfrac{x+2}{2(x+3)}$

VA: $x = -3$

holes: none

HA: $y = \frac{1}{2}$

Zeros: $x = -2$

13) $f(x) = \dfrac{2x^2 + 10x + 12}{x^2 + 3x + 2} = \dfrac{2(x+5)(x+1)}{(x+2)(x+1)}$

VA: $x = -2$

hole: $(-1, 8)$

HA: $y = 2$

Zeros: $x = -5$

y coordinate

$\dfrac{2(-1+5)}{-1+2} = \dfrac{8}{1}$

14) $f(x) = \dfrac{3}{x - 2}$

VA: $x = 2$

holes none

HA: $y = 0$

Zeros: none

2. $y = \dfrac{x^2 + 5x + 6}{x^2 - 9}$ $\dfrac{(x+5)(x+1)}{(x+3)(x-3)}$

VA: $x = 3, -3$

holes: none

HA: $y = 1$

Zeros: $x = -5, -1$

3. $y = \dfrac{x^2 - 4}{3x^2 - 15x + 18}$ $\dfrac{(x+2)(x-2)}{3(x-3)(x-2)}$

VA: $x = 3$

holes: $(2, \frac{-4}{3})$

HA: $y = \frac{1}{3}$

Zeros: $x = -2$

y coordinate

$\dfrac{2+2}{3(2-3)}$

$= \dfrac{-4}{3}$