Lesson 1.21 - Rational Function Attributes

Learning Objectives: SWBAT

1. Identify discontinuities in Rational function graphs as Vertical Asymptotes or holes and determine their location given the function's equation
2. Determine the horizontal Asymptotes of a rational function
3. Determine the zeros of a rational function

What are *Discontinuities*?

- Discontinuities are "gaps" in what would otherwise be a continuous graph. The two types of discontinuities are Vertical Asymptotes and Holes.

What is an *Asymptote*?

- An Asymptote represents the *end behavior* of a curved graph as it approaches either +/- infinity. The graph approaches the asymptote but NEVER touches/crosses it. Asymptotes can vertical, horizontal or oblique (slanted)

What is a *Vertical Asymptote*?

- A Vertical Asymptote is a vertical **line of discontinuity** on the graph. It represents a x values where the y value is undefined.

What is a *hole*?

- A Hole is a **point of discontinuity** on the graph. Similar to vertical asymptotes above, holes represent x value(s) where the y value is undefined.

How do we find Vertical Asymptotes and Holes?

- A Vertical Asymptote is the result when the factor that causes the denominator to equal zero is NOT cancelled while simplifying
- A Hole is the result when the factor that causes the denominator to equal 0 also exists in the numerator (and thereby can be cancelled out)

Example: Identify the number of discontinuities present in the rational function below. Identify the location of any vertical Asymptotes and the coordinates of any holes

$f(x) = \dfrac{x-4}{x^2-16}$ factor the denominator ⟶ $\dfrac{(x-4)}{(x+4)(x-4)}$ notice how x = 4 cancels

- There is a vertical Asymptote at x = -4
- There is a hole at x = 4. This is the x coordinate of the hole

To find the y coordinate of the hole:

> Simplify (cancel) the expression above and plug in the x coordinate of the hole

$$\frac{(x-4)}{(x+4)(x-4)} \longrightarrow \frac{1}{(x+4)} \longrightarrow \frac{1}{((4)+4)} \longrightarrow \frac{1}{8}$$

> The coordinate of the hole is $\left(4, \frac{1}{8}\right)$

Your Turn Determine the locations of any vertical asymptotes and the corrdinates of any hole of the following function

$$f(x) = \frac{x^2 - 9}{x^2 + 4x + 3}$$

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How do we find Horizontal Asymptotes?

- The existence and location of a horizontal asymptote depends on the one of the following conditions:
  - > If the degree of the numerator is less than the degree of the denominator, then a a horizontal asymptote exists at y = 0
  - > If the degree of the numerator is greater than the degree of the denominator, then a horizontal asymptote does not exist
  - > If the degree of the numerator and denominator are the same then a horizontal asymptote will exist at the following line:

$$y = \frac{\text{lead coefficient of numerator}}{\text{lead coefficient of denominator}}$$

Example Let $y = \dfrac{3 - 2x - x^2}{x^2 + x - 2}$. Identify all asymptotes and holes in the graph.

SOLUTION

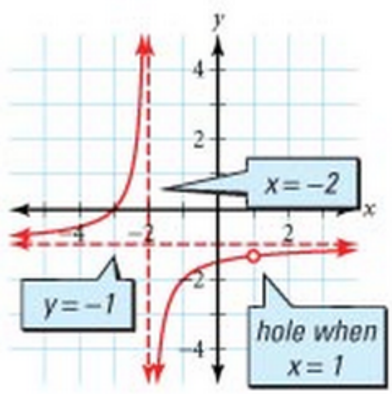
Factor the numerator and denominator.

$$y = \frac{3 - 2x - x^2}{x^2 + x - 2} = \frac{-(x^2 + 2x - 3)}{(x-1)(x+2)} = \frac{-(x+3)\cancel{(x-1)}}{\cancel{(x-1)}(x+2)}$$

Because $x - 1$ is a factor of *both* the numerator and the denominator, the graph has a hole when $x = 1$.

Because $x + 2$ is a factor of *only* the denominator, there is a vertical asymptote at $x = -2$.

Because the degree of the numerator equals the degree of the denominator, there is a horizontal asymptote at $y = \frac{-1}{1} = -1$.



How do we find zeros (x intercepts)?

- Look to the numerator. Whatever x value makes the numerator zero is the x value of the x intercept
- In the example above, the coordinates of the x intercept is (-3, 0)

Your Turn - Find any vertical asymptotes, holes, horizontal asymptotes and zeros for the following function:

$$f(x) = \frac{x^3 + 3x^2}{x^2 + 2x - 3}$$

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For each function: Find any vertical asymptotes, holes, horizontal asymptotes and zeros for the following function:

1) $f(x) = \dfrac{1}{3x^2 + 3x - 18}$

2) $f(x) = \dfrac{x - 2}{x - 4}$

3) $f(x) = \dfrac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

4) $f(x) = \dfrac{x^2 + x - 6}{-4x^2 - 16x - 12}$

5) $f(x) = -\dfrac{4}{x^2 - 3x}$

6) $f(x) = \dfrac{x - 4}{-4x - 16}$

7) $f(x) = \dfrac{x + 4}{-2x - 6}$

8) $f(x) = \dfrac{x^3 - 9x}{3x^2 - 6x - 9}$

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9) $f(x) = \dfrac{3x^2 - 12x}{x^2 - 2x - 3}$

10) $f(x) = \dfrac{x^3 - 16x}{-4x^2 + 4x + 24}$

11) $f(x) = \dfrac{x^2 + 2x}{-4x + 8}$

12) $f(x) = \dfrac{x + 2}{2x + 6}$

13) $f(x) = \dfrac{2x^2 + 10x + 12}{x^2 + 3x + 2}$

14) $f(x) = \dfrac{3}{x - 2}$

2. $y = \dfrac{x^2 + 5x + 6}{x^2 - 9}$

3. $y = \dfrac{x^2 - 4}{3x^2 - 15x + 18}$