Lesson 1.22 - Slant (Oblique) Asymptotes

Learning Objectives: SWBAT

1. Identify the conditions in which a slant asymptote exists for a rational function
2. Write the equation of the slant asymptote for a given rational function

Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, the graph of the function has a **slant** (or **oblique**) **asymptote.** For example, the graph of

$$f(x) = \frac{x^2 - x}{x + 1}$$

has a slant asymptote, as shown in Figure 2.62. To find the equation of a slant asymptote, use long division. For instance, by dividing $x + 1$ into $x^2 - x$, you have

$$f(x) = \frac{x^2 - x}{x + 1} = \underbrace{x - 2}_{\substack{\text{Slant asymptote} \\ (y = x - 2)}} + \frac{2}{x + 1}.$$

As x increases or decreases without bound, the remainder term $2/(x + 1)$ approaches 0, so the graph of f approaches the line $y = x - 2$, as shown in Figure 2.62.

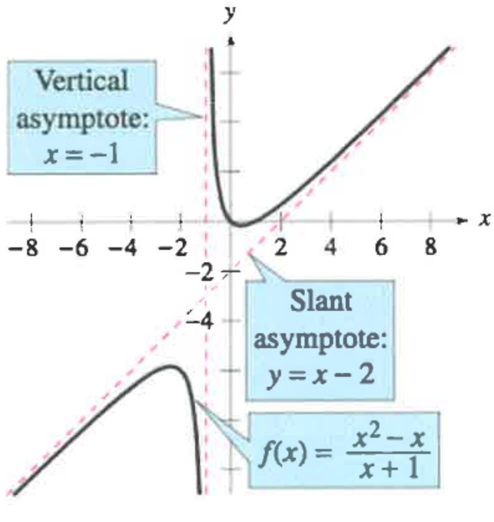

Figure 2.62

Your Turn - Determine the equation of the slant asymptote for the function below:

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

$x - 1 \overline{)\, x^2 - x - 2}$ with quotient x; $x^2 - x$; remainder -2 $= x - \frac{2}{x-1}$

$y = x$

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Practice: Determine whether or not a slant asymptote exists for the functions below. If it does exist, write its equation/location

43. $f(x) = \frac{2x^2 + 1}{x}$

$y = 2x$

44. $g(x) = \frac{1 - x^2}{x}$

$y = -x$

45. $h(x) = \frac{x^2}{x - 1}$

$y = x + 1$

46. $f(x) = \frac{x^3}{x^2 - 1}$

$y = x$

47. $g(x) = \frac{x^3}{2x^2 - 8}$

$y = \frac{1}{2}x$

48. $f(x) = \frac{x^2 - 1}{x^2 + 4}$

No slant Asymptote

49. $f(x) = \frac{x^3 + 2x^2 + 4}{2x^2 + 1}$

$y = \frac{1}{2}x + 1$

50. $f(x) = \frac{2x^2 - 5x + 5}{x - 2}$

$y = 2x - 1$

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Practice: Determine whether or not a slant asymptote exists for the functions below. If it does exist, write its equation/location

55. $y = \dfrac{2x^2 + x}{x + 1}$

$y = 2x - 1$

56. $y = \dfrac{x^2 + 5x + 8}{x + 3}$

$y = x + 2$

57. $y = \dfrac{1 + 3x^2 - x^3}{x^2}$

$y = -x + 3$

58. $y = \dfrac{12 - 2x - x^2}{2(4 + x)}$

$y = -\frac{1}{2}x + 1$

63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

$y = 2x - 7$

64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$

$y = 2x + 7$