Lesson 1.22 - Slant (Oblique) Asymptotes

Learning Objectives: SWBAT

1. Identify the conditions in which a slant asymptote exists for a rational function
2. Write the equation of the slant asymptote for a given rational function

Consider a rational function whose denominator is of degree 1 or greater. If the degree of the numerator is exactly *one more* than the degree of the denominator, the graph of the function has a **slant** (or **oblique**) **asymptote.** For example, the graph of

$$f(x) = \frac{x^2 - x}{x + 1}$$

has a slant asymptote, as shown in Figure 2.62. To find the equation of a slant asymptote, use long division. For instance, by dividing $x + 1$ into $x^2 - x$, you have

$$f(x) = \frac{x^2 - x}{x + 1} = \underbrace{x - 2}_{\text{Slant asymptote } (y = x - 2)} + \frac{2}{x + 1}.$$

As x increases or decreases without bound, the remainder term $2/(x + 1)$ approaches 0, so the graph of f approaches the line $y = x - 2$, as shown in Figure 2.62.

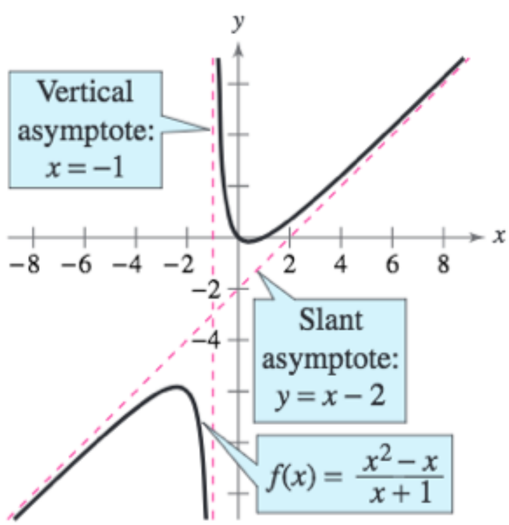

Figure 2.62

Your Turn - Determine the equation of the slant asymptote for the function below:

$$f(x) = \frac{x^2 - x - 2}{x - 1}$$

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Practice: Determine whether or not a slant asymptote exists for the functions below. If it does exist, write its equation/location

43. $f(x) = \dfrac{2x^2 + 1}{x}$

44. $g(x) = \dfrac{1 - x^2}{x}$

45. $h(x) = \dfrac{x^2}{x - 1}$

46. $f(x) = \dfrac{x^3}{x^2 - 1}$

47. $g(x) = \dfrac{x^3}{2x^2 - 8}$

48. $f(x) = \dfrac{x^2 - 1}{x^2 + 4}$

49. $f(x) = \dfrac{x^3 + 2x^2 + 4}{2x^2 + 1}$

50. $f(x) = \dfrac{2x^2 - 5x + 5}{x - 2}$

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Practice: Determine whether or not a slant asymptote exists for the functions below. If it does exist, write its equation/location

55. $y = \dfrac{2x^2 + x}{x + 1}$

56. $y = \dfrac{x^2 + 5x + 8}{x + 3}$

57. $y = \dfrac{1 + 3x^2 - x^3}{x^2}$

58. $y = \dfrac{12 - 2x - x^2}{2(4 + x)}$

63. $f(x) = \dfrac{2x^3 - x^2 - 2x + 1}{x^2 + 3x + 2}$

64. $f(x) = \dfrac{2x^3 + x^2 - 8x - 4}{x^2 - 3x + 2}$