Lesson 1.23 - Solving Rational Equations

Learning Objectives: SWBAT

1. Solve Rational Equations
2. Check for and identify any extraneous solutions

Making a connection:

- The process of solving rational equations is similar to other equations we have solved. We just need to remember all of the basic rules of solving equations
  - > Ultimately, we want to find solve for x (or whatever variable is in the problem)
  - > In the process of solving, whatever we do to one side of the equals sign we do to the other side of the equals sign (keeping balance)
  - > The process of solving involves undoing the operation that is in front of us (for example, if we see division, we undo it by multiplying)We need to remember order of operations (PEMDAS)
- The "wrinkle" with rational equations is that we need to remember rules of operating with fractions (things such as common denominators when adding/ subtracting and multiplying by a reciprocal to "get rid of" the fraction all together
- The first step for solving these is often to "get rid of" the denominator because it is easier to work numbers that are not fractions
- There will also be times when the process will lead us to creating two ratios that are equal to each other. In these situations, we need to remember the rule "cross multiply" to solve:

Example 1: Solve the following Ratio:

$$\frac{v}{12}=\frac{10}{2} \xrightarrow{\text{cross multiply}} 2v = 120 \longrightarrow v = 60$$

- The same idea can be applied to rational expressions that equal to each other

Example 2: Solve the following ratio

$$\frac{(x+5)}{(x+8)}=\frac{(x+7)}{(x+1)} \xrightarrow{\text{cross multiply}} (x+5)(x+1) = (x+7)(x+8)$$

$$\xrightarrow{\text{FOIL}} x^2 + 6x +5 = x^2 + 15x +56 \xrightarrow{\text{Solve for x}} -9x = 51 \longrightarrow x = -51/9 \overset{\text{Simplify}}{=} -\frac{17}{3}$$

- Sometimes there will be more than one solution to an equation

Example 3: Solve $\frac{x}{x-6}=\frac{1}{x-4}$.

SOLUTION

$$x(x-4)=1(x-6)$$
$$x^2-4x=x-6$$
$$x^2-5x+6=0$$
$$(x-2)(x-3)=0$$
$$x=2 \quad or \quad x=3$$

CHECK

Let $x=2$.

$$\frac{x}{x-6}=\frac{1}{x-4}$$
$$\frac{2}{2-6}\stackrel{?}{=}\frac{1}{2-4}$$
$$-\frac{1}{2}=-\frac{1}{2} \quad \text{True}$$

Let $x=3$.

$$\frac{x}{x-6}=\frac{1}{x-4}$$
$$\frac{3}{3-6}\stackrel{?}{=}\frac{1}{3-4}$$
$$-1=-1 \quad \text{True}$$

The solutions are 2 and 3.

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Sometimes solving a rational equation introduces *extraneous solutions*. An **extraneous solution** is a solution to a resulting equation that is not a solution to the original equation. Therefore, it is important to check your answers, as shown in Example 3.

Example 4: Solve $\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2-9}$

SOLUTION Multiply each side of the equation by the LCD, $(x-3)(x+3)$, or $x^2 - 9$.

$$\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2-9}, \text{ where } x \neq 3 \text{ and } x \neq -3$$

$$\frac{x}{x-3}(x+3)(x-3) + \frac{2x}{x+3}(x+3)(x-3) = \frac{18}{x^2-9}(x+3)(x-3)$$

$$x(x+3) + 2x(x-3) = 18$$

$$x^2 + 3x + 2x^2 - 6x = 18$$

$$3x^2 - 3x - 18 = 0$$

$$3(x^2 - x - 6) = 0$$

$$3(x-3)(x+2) = 0$$

$$x = 3 \quad or \quad x = -2$$

CHECK

Since $x = 3$ is an excluded value of x in the original equation, it is an extraneous solution. Check $x = -2$.

$$\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2-9}$$

$$\frac{-2}{-2-3} + \frac{2(-2)}{-2} + 3 \stackrel{?}{=} \frac{18}{(-2)^2-9}$$

$$-3\frac{3}{5} = \frac{-18}{5} \quad \text{True}$$

Thus, the only solution is $x = -2$.

Your Turn Solve $\frac{x}{x-2} + \frac{x}{x-3} = \frac{3}{x^2-5x+6}$

$(x-2)(x-3)$

$$\frac{x(x-3) + x(x-2) - 3}{(x-2)(x-3)} = 0 \rightarrow$$

$x^2 - 3x + x^2 - 2x - 3 = 0$

$2x^2 - 5x - 3 = 0$

$(2x^2 - 6x)(x - 3) = 0$

$2x(x-3) + 1(x-3) = 0$

$(2x+1)(x-3) = 0$

$\boxed{x = -\frac{1}{2}, 3}$

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Practice: **Solve each equation. Remember to check for extraneous solutions.**

1) $\frac{1}{6k^2} = \frac{1}{3k^2} - \frac{1}{k}$

$\frac{-1 + (2) - 6k}{6k^2} = 0$

$-6k + 1 = 0$

$\boxed{k = \frac{1}{6}}$

2) $\frac{1}{n^2} + \frac{1}{n} = \frac{1}{2n^2}$

$\frac{2 + 2n - 1}{2n^2} = 0 \rightarrow 2n + 1 = 0$

$\boxed{n = -\frac{1}{2}}$

3) $\frac{1}{6b^2} + \frac{1}{6b} = \frac{1}{b^2}$

$\boxed{b = 5}$

4) $\frac{b+6}{4b^2} + \frac{3}{2b^2} = \frac{b+4}{2b^2}$

$\frac{b+6+3(2)-2(b+4)}{4b^2} = 0$

$b + 6 + 6 - 2b - 8 = 0$

$-b + 4 = 0$ $\boxed{b = 4}$

5) $\frac{1}{x} = \frac{6}{5x} + 1$

$\frac{6 + (5x) - (5)}{5x} = 0 \rightarrow 5x + 1 = 0$

$\boxed{x = -\frac{1}{5}}$

6) $\frac{1}{6x^2} = \frac{1}{2x} + \frac{7}{6x^2}$

$\boxed{x = -2}$

7) $\frac{1}{v} + \frac{3v+12}{v^2-5v} = \frac{7v-56}{v^2-5v}$

$\boxed{x = 21}$

8) $\frac{1}{m^2-m} + \frac{1}{m} = \frac{5}{m^2-m}$

$\boxed{m = 5}$

9) $\frac{1}{n-8} - 1 = \frac{7}{n-8}$

$\boxed{n = 2}$

10) $\frac{1}{r-2} + \frac{1}{r^2-7r+10} = \frac{6}{r-2}$

$(r-2)(r-5)$

$\frac{(r-5) + 1 - 6(r-5)}{(r-2)(r-5)} =$

$r - 5 + 1 - 6r + 30 = 0$

$26 - 5r = 0$

$-5r = -26$

$\boxed{r = \frac{26}{5}}$

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Practice: **Solve each equation. Remember to check for extraneous solutions.**

11) $1 = \frac{v+2}{v-4} + \frac{7v-42}{v-4}$

$\boxed{v = \frac{36}{7}}$

12) $\frac{r-4}{5r} = \frac{1}{5r} + 1$

$\boxed{r = \frac{-5}{4}}$

13) $1 + \frac{x^2-5x-24}{3x} = \frac{x-6}{3x}$

$\frac{3x + x^2 - 5x - 24 - x + 6}{3x} = 0$

$x^2 - 3x - 18 = 0 \rightarrow (x+3)(x-6)$

$\boxed{x = -3, 6}$

14) $1 = \frac{1}{x^2+2x} + \frac{x-1}{x}$

$\boxed{x = -1}$

15) $\frac{n+5}{n+8} = 1 + \frac{6}{n+1}$

$\boxed{n = \frac{-17}{3}}$

16) $\frac{r+5}{r^2-2r} - 1 = \frac{1}{r^2-2r}$

$\boxed{r = 4, -1}$

17) $\frac{1}{x^2-5x} = \frac{x+7}{x} - 1$

$\boxed{x = \frac{36}{7}}$

18) $\frac{a-2}{a+3} - 1 = \frac{3}{a+2}$

$\boxed{a = \frac{-19}{8}}$

19) $\frac{p+5}{p^2+p} = \frac{1}{p^2+p} - \frac{p-6}{p+1}$

$6p(p+1)$ $\quad p(p+1)$

$\frac{1 - p(p-6) - (p+5)}{p(p+1)} \rightarrow 1 - p^2 + 6p - p - 5$

$-p^2 + 5p - 4$

$-1(p^2 - 5p + 4) \rightarrow (p-4)(p-1)$ $\boxed{p = 4, 1}$

20) $\frac{5}{n^3+5n^2} = \frac{4}{n+5} + \frac{1}{n^2}$

$\boxed{n = -1/4}$