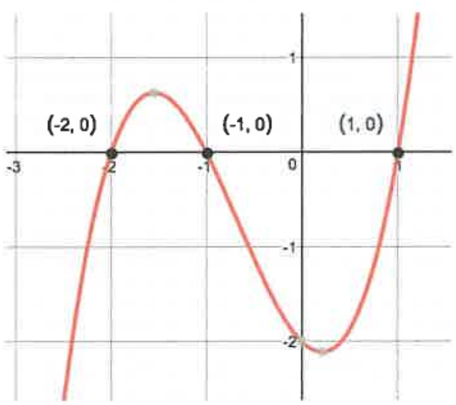
Lesson 1.24 - Polynomial Inequalities

Learning Objectives: SWBAT
1. Explain the difference between the solution to a polynomial equation versus a polynomial inequality
2. Solve polynomial equations, quantify the solution using interval notation

Making a connection - The difference between equations and inequalities
- In Unit 1A, we solved polynomial equations by finding the values that made y = 0
- These solutions were *single points* where the graph crossed the x axis
- From Algebra 2, we know that solutions simple inequalities (such as x > 1) are an *infinite number of points*
- Solutions to polynomial inequalities work similarly. They are *intervals* upon which the values on the graph are greater than or less than zero.
- The key to understanding this idea is in knowing how the graph works. Below is a comparison of a polynomial equation and inequality for the same function

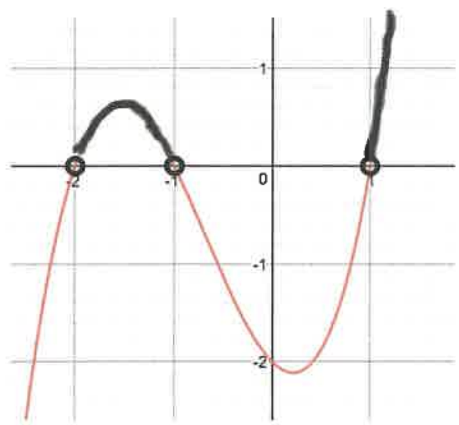
Solutions to equation

$$x^3 + 2x^2 - x - 2 = 0$$



Solutions to inequality

$$x^3 + 2x^2 - x - 2 > 0$$



Things to notice:
- The solution to the inequality are all of the points with a y > 0. If the inequality was a "less than", then the solution would be all of the points where y < 0. Because there is an infinite number of points between each boundary, there are an infinite number of solutions to the inequality (similar to a simple inequality)
- Open circles signify that the zeros themselves are NOT included in the solution. If the inequality was a ≥ or ≤ then these circles would be closed
- The solutions to the equation set the boundaries for the intervals of solutions to the inequality

How to write the solution:
- To write the solution, we must use interval notation, because the parts of the solution are on "intervals" on the x axis
- The first interval is from -2 to -1, the second interval is from 1 to + ∞
- We would write the solution using interval notation (-2, -1) U (1, ∞)
  - > Remember: If the inequality were a ≥ or ≤ then then we would use brackets [ ]
- For the function $\mathbf{x^3 + 2x^2 - x - 2 < 0}$, the solution would be intervals (-∞ , -2) U (-1, 1)

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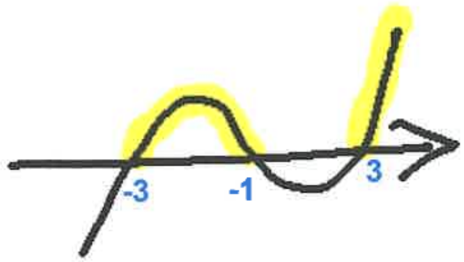
Example: Solve the following polynomial inequality: $\mathbf{x^3 + x^2 - 9x + 3 > 12}$

$x^3 + x^2 - 9x - 9 > 0$	Step 1 - Make right side = 0
$(x + 3)(x - 3)(x + 1) > 0$	Step 2 - Factor
$x = 3, x = -3, x = -1$	Step 3 - Determine zeros and multiplicities

- From here, you have two choices as to how to finish

1. Do a rough sketch of the polynomial using end behavior, multiplicity and the zeros as we learned in unit 1A. The rough sketch of the polynomial would look like this:



2. Draw a number line with the zeros as boundaries for the intervals (see below)

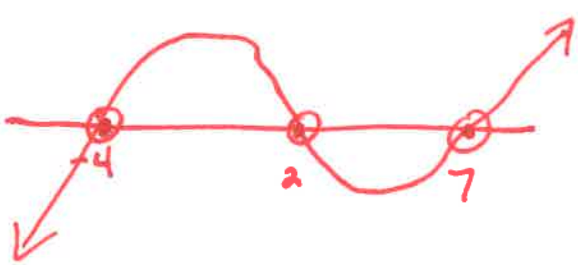
Pick a "test point" from each interval and evaluate (plug it into) the function to determine if the result is positive or negative:
* Evaluate f(-4), the result is negative, therefore numbers less than -3 ARE NOT part of the solution
* Evaluate f(-2), the result is positive, therefore numbers in between -3 and -1 ARE part of the solution
* Evaluate f(0), the result is negative, therefore numbers in between -1 and - ARE part of the solution
* Evaluate f(4), the result is positive, therefore numbers in between greater than 3 ARE part of the solution

Label the positive negative areas above/below the number line

3. The solutions are all intervals where f(x) is positive. The zeros are NOT included in the solution.

« **Writing in interval notation, the solution is (-3, -1) U (3, ∞)**

Your Turn: Solve the inequality $(x+4)(x-2)(x-7) > 0$



$(-4, 2) \cup (7, \infty)$

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Practice:

Solve each inequality.

1) $(x-4)(x+3)<0$

$(-3, 4)$

2) $(x-4)(x+1)\geq 0$

$(-\infty, -1] \cup [4, \infty)$

3) $(x-1)(3x-4)\geq 0$

$(-\infty, 1] \cup [\frac{4}{3}, \infty)$

4) $(x+8)(x+2)(x-3)\geq 0$

$[-8, -2] \cup [3, \infty)$

5) $x^2+5x+4\leq 0$

$[-4, -1]$

6) $x^2-14x+49\geq 0$

$(-\infty, \infty)$

7) $x^2-4x-32>0$

$(-\infty, -4) \cup (8, \infty)$

8) $x^2+16x+24>6x$

$(-\infty, -6) \cup (-4, \infty)$

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Practice:

9) $(x+5)(x-2)(x-1)(x+1)<0$

$(-5,-1)\cup(1,2)$

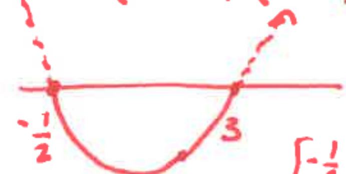
10) $(x+8)^2(x+5)(x+7)^2\geq 0$

$\{-8\}\cup\{-7\}\cup[-5,\infty)$

11. $2x^2-3\leq x$

$2x^2-x-3\leq 0$

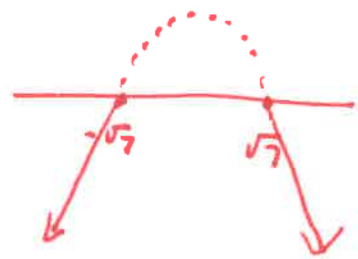
$(2x+1)(x-3)$



$[-\frac{1}{2},3]$

12. $7-x^2\leq 0$

$0\leq x^2-7 \rightarrow$ zeros $\pm\sqrt{7}$



$(-\infty,-\sqrt{7}]\cup[\sqrt{7},\infty)$

13. $x^4-5x^2\leq -4$

$x^4-5x^2+4\leq 0$

$(x^2-4)(x^2-1)$

$(x+2)(x-2)(x+1)(x-1)$

$[-2,-1]\cup[1,2]$

14. $x^5+9x\geq 10x^3$

$x^5-10x^3+9x\geq 0$

$x(x^4-10x^2+9)\geq 0$

$x(x^2-9)(x^2-1)\geq 0$

$x(x+3)(x-3)(x+1)(x-1)\geq 0$

$[-\infty,-3]\cup[-1,0]\cup[1,3]$

15. $x^3-11x^2-8x+88\geq 0$

$x^2(x-11)-8(x-11)\geq 0$

$(x^2-8)(x-11)\geq 0$

$[-\sqrt{8},\sqrt{8}]\cup[11,\infty)$

16. $x^4-13x^2+36\leq 0$

$(x^2-9)(x^2-4)\leq 0$

$(x+3)(x-3)(x+2)(x-2)\leq 0$

$[-3,-2]\cup[2,3]$