Lesson 1.25 - Rational Inequalities

Learning Objectives: SWBAT

1. Solve rational inequalities quantifying the solution using interval notation

What is a rational inequality?

- In essence, it is the same idea as solving polynomial inequalities
  - > Solutions will always be an INTERVAL of values in which the polynomial is > or < 0
- The difference is that for rational inequalities, we not only define our intervals based on zeros, but also on the excluded values:

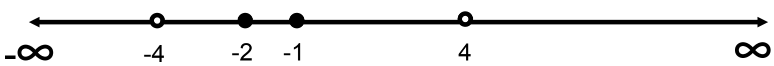
Example 1: Solve the following rational inequality: $\frac{x^2+3x+2}{x^2-16} \geq 0$

> Step 1: Factor everything $\frac{x^2+3x+2}{x^2-16} = \frac{(x+2)(x+1)}{(x+4)(x-4)}$

> Step 2: Determine the zeros and excluded values

- – zeros: x = -2, x = -1; Excluded values x ≠ -4, x ≠ 4
- – These values will create our domain intervals

> Step 3: Create a number line with each value above noted



- – Notice: The excluded values will ALWAYS be an open circle because there it holes/vertical asymptotes are undefined on the graph.

> Step 4: Pick a value within each interval and evaluate the function. If the result is positive then, the interval is included in the solution. If the result is negative, then the interval is NOT included in the solution

- – Interval 1: f(-5) = + 4/3 ------> (-∞, -4) is part of the solution
- – Interval 2: f(-3) = - 3/7 ------> (-4, -1] is not part of the solution
- – Interval 3: f(-1.5) = +.818 ------> [-2, -1] is part of the solution
- – Interval 4: f(0) = -1/8 ------> [-1, 4) is not part of the solution
- – Interval 5: f(5) = +14/3 ------> (4, ∞) is part of the solution

> Step 5: Write your final answer using the intervals that are part of the solution. In this problem, we are looking for the intervals that are positive:

(-∞, -4) U [-2, -1] U (4, ∞)

- – Notice: As with polynomial inequalities, be careful note which intervals use [ ] vs. ( ). make sure you look at the inequality sign to determine if the zeros are open/closed circles.

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Example 2: In this example, we need to combine the fractions using common denominators before we determine the interval boundaries:

$$\frac{7-x}{x+1}-\frac{4-x}{x+3}>0$$

$$\frac{(7-x)(x+3)-(4-x)(x+1)}{(x+1)(x+3)}>0$$

$$\frac{7x+21-x^2-3x-\left(4x+4-x^2-x\right)}{(x+1)(x+3)}>0$$

$$\frac{21+4x-x^2-\left(4+3x-x^2\right)}{(x+1)(x+3)}>0$$

$$\frac{21+4x-x^2-4-3x+x^2}{(x+1)(x+3)}>0$$

$$\frac{x+17}{(x+1)(x+3)}>0$$

The values $x=-17$, $x=-3$, and $x=-1$ split the real numbers into the following intervals.

$(-\infty,-17)$, $(-17,-3)$, $(-3,-1)$, $(-1,\infty)$

Interval	Test value (x)	Value of $\frac{x+17}{(x+1)(x+3)}$
$(-\infty,-17)$	-18	$\frac{-18+17}{(-18+1)(-18+3)}<0$
$(-17,-3)$	-4	$\frac{-4+17}{(-4+1)(-4+3)}>0$
$(-3,-1)$	-2	$\frac{-2+17}{(-2+1)(-2+3)}<0$
$(-1,\infty)$	0	$\frac{0+17}{(0+1)(0+3)}>0$

$$\boxed{(-17,-3)\cup(-1,\infty)}$$

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Practice:

Solve each inequality.

1) $\frac{x-7}{x-1} < 0$

2) $\frac{x+5}{x-4} \le 0$

3) $\frac{x+32}{x+6} \le 3$

4) $\frac{x+68}{x+8} \ge 5$

5) $\frac{(x+3)(x+5)}{x+2} \ge 0$

6) $\frac{x+6}{x^2-5x-24} \ge 0$

7) $-\frac{10}{x-5} \ge -\frac{11}{x-6}$

8) $-\frac{3}{x+7} \le -\frac{4}{x+8}$

9) $-\frac{7}{x+5} \le -\frac{8}{x+6}$

10) $\frac{(x+7)(x-3)}{(x-5)^2} > 0$

Critical thinking question:

11) Write a rational inequality with the solution: $(-2,-1)\cup(1,\infty)$

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Practice:

Solve each inequality.

11. $\dfrac{2x}{x-2} \leq 3$

12. $\dfrac{1}{x+2} \geq \dfrac{1}{3}$

13. $\dfrac{1}{4} < \dfrac{7}{7-x}$

14. $\dfrac{x+2}{x+5} \geq 1$

15. $\dfrac{3}{x-2} \leq \dfrac{3}{x+3}$

16. $x - \dfrac{10}{x-1} \geq 4$