Lesson 1.26 - Absolute Value Equations

Learning Objectives: SWBAT

1. Solve Absolute Value Equations and identify extraneous solutions

What is Absolute Value?

- Absolute Value is the (positive) distance away from zero of a number on the number line

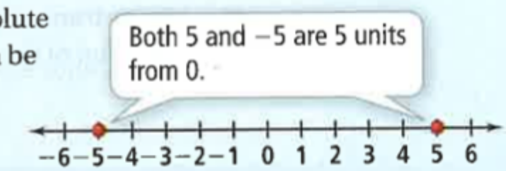
Essential Understanding An absolute value quantity is nonnegative. Since opposites have the same absolute value, an absolute value equation can have two solutions.

take note

Key Concept **Absolute Value**

Definition	Numbers	Symbols
The **absolute value** of a real number x, written $\lvert x \rvert$, is its distance from zero on the number line.	$\lvert 4 \rvert = 4$ $\lvert -4 \rvert = 4$	$\lvert x \rvert = x$, if $x \geq 0$ $\lvert x \rvert = -x$, if $x < 0$

An absolute value equation has a variable within the absolute value sign. For example, $\lvert x \rvert = 5$. Here, the value of x can be 5 or -5 since $\lvert 5 \rvert$ and $\lvert -5 \rvert$ both equal 5.



What is the process for solving AV Equations?

1. Isolate the Absolute Value term as if you were solving for a single variable
2. Once the AV term is isolated, break it up into two equations, one for the positive solution, and another for the negative. Then, solve for each to get two solutions

Example 1 **What is the solution of $3\lvert x + 2 \rvert - 1 = 8$? Graph the solution.**

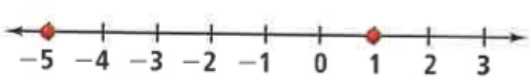
$3\lvert x + 2 \rvert - 1 = 8$

$3\lvert x + 2 \rvert = 9$ — Add 1 to each side.

Isolated AV term → $\lvert x + 2 \rvert = 3$ — Divide each side by 3.

$x + 2 = 3$ or $x + 2 = -3$ — Rewrite as two equations.

$x = 1$ or $x = -5$ — Subtract 2 from each side of both equations.



Check $3\lvert (1) + 2 \rvert - 1 \stackrel{?}{=} 8$ $\quad$ $3\lvert (-5) + 2 \rvert - 1 \stackrel{?}{=} 8$

$3\lvert 3 \rvert - 1 \stackrel{?}{=} 8$ $\quad$ $3\lvert -3 \rvert - 1 \stackrel{?}{=} 8$

$8 = 8$ ✔ $\quad$ $8 = 8$ ✔

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When will we get extraneous solutions?

Distance from 0 on the number line cannot be negative. Therefore, some absolute value equations, such as $|x| = -5$, have no solution. It is important to check the possible solutions of an absolute value equation. One or more of the possible solutions may be *extraneous.*

An **extraneous solution** is a solution derived from an original equation that is *not* a solution of the original equation.

Example 2 **What is the solution of $|3x + 2| = 4x + 5$? Check for extraneous solutions.**

$$|3x + 2| = 4x + 5$$

$3x + 2 = 4x + 5$ or $3x + 2 = -(4x + 5)$ Rewrite as two equations.

$-x = 3$ | $3x + 2 = -4x - 5$ Solve each equation.

$7x = -7$

$x = -3$ or $x = -1$

Check $|3(-3) + 2| \stackrel{?}{=} 4(-3) + 5$ $|3(-1) + 2| \stackrel{?}{=} 4(-1) + 5$

$|-9 + 2| \stackrel{?}{=} -12 + 5$ $|-3 + 2| \stackrel{?}{=} -4 + 5$

$|-7| \neq -7$ ✘ $|-1| = 1$ ✔

Since $x = -3$ does not satisfy the orginal equation, -3 is an extraneous solution. ' only solution to the equation is $x = -1$.

Your Turn #1 What is the solution of $2|x + 9| + 3 = 7$? Graph the solution.

Your Turn #2 What is the solution of $|5x - 2| = 7x + 14$? Check for extraneous solutions.

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Practice - Solve each equation, check/identify any extraneous solutions

10. $|3x| = 18$

11. $|-4x| = 32$

12. $|x - 3| = 9$

13. $2|3x - 2| = 14$

14. $|3x + 4| = -3$

15. $|2x - 3| = -1$

16. $|x + 4| + 3 = 17$

17. $|y - 5| - 2 = 10$

18. $|4 - z| - 10 = 1$

19. $|x - 1| = 5x + 10$

20. $|2z - 3| = 4z - 1$

21. $|3x + 5| = 5x + 2$

22. $|2y - 4| = 12$

23. $3|4w - 1| - 5 = 10$

24. $|2x + 5| = 3x + 4$