Lesson 1.27 - Absolute Value Inequalities

Learning Objectives: SWBAT

1. Solve Absolute Value Inequalities and graph the solution on a number line

Making a connection

- In lesson 1.26 we solved AB equations. As with other equations we have solved, the nature of the solution were single point(s) that satisfied the equation
- As with other inequalities we have solved, the nature of the solutions to AV inequalities is an infinite "range" of points.
- The idea/process of solving for the "boundary" values is exactly the same in lesson 1.26. The difference that the < > signs determine where the "range" of solutions is located

Example 1 Solving the Absolute Value Inequality $|A| < b$

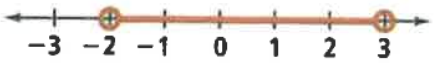
What is the solution of $|2x - 1| < 5$? Graph the solution.

$|2x - 1| < 5$

$-5 < 2x - 1 < 5$ $2x - 1$ is between -5 and 5.

$-4 < 2x < 6$ Add 1 to each part.

$-2 < x < 3$ Divide each part by 2.



- Things to notice:
  - > When the problem is a LESS THAN problem, the solutions are between the two boundaries
  - > Closed circles would be used if the problem uses a ≤ or ≥ sign

Example 2 Solving the Absolute Value Inequality $|A| \geq b$

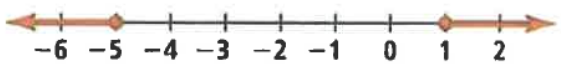
What is the solution of $|2x + 4| \geq 6$? Graph the solution.

$|2x + 4| \geq 6$

$2x + 4 \leq -6$ or $2x + 4 \geq 6$ Rewrite as a compound inequality.

$2x \leq -10$ | $2x \geq 2$ Subtract 4 from each side of both inequalities.

$x \leq -5$ or $x \geq 1$ Divide each side of both inequalities by 2.



- Things to notice:
  - > When the problem is a GREATER THAN problem, the solutions are outside the two boundaries
  - > Open circles would be used if the problem uses a < or > sign

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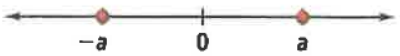
take note

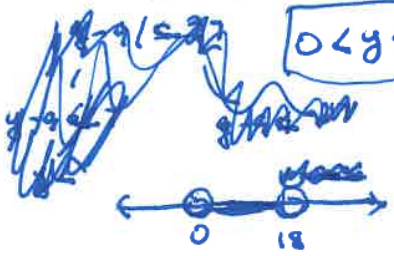
Concept Summary Solutions of Absolute Value Statements

Symbols	Definition	Graph
$\lvert x\rvert = a$	The distance from x to 0 is a units.	 $x = -a$ or $x = a$
$\lvert x\rvert < a$ $(\lvert x\rvert \leq a)$	The distance from x to 0 is less than a units.	 $-a < x < a$ $x > -a$ and $x < a$
$\lvert x\rvert > a$ $(\lvert x\rvert \geq a)$	The distance from x to 0 is greater than a units.	 $x < -a$ or $x > a$

Practice **Solve each inequality. ~~Graph the solution.~~**

25. $3|y - 9| < 27$

$0 < y < 18$



26. $|6y - 2| + 4 < 22$

$|6y - 2| < 18$

$6y - 2 < 18$ $\quad 6y - 2 > -18$

$y < \frac{10}{3}$ and $y < -\frac{8}{3}$

27. $|3x - 6| + 3 < 15$

$-2 < x < 6$

28. $\frac{1}{4}|x - 3| + 2 < 1$

$\frac{1}{4}|x-3| < -1$

$|x-3| < -4$

No solution

AV can't be negative

29. $4|2w + 3| - 7 \leq 9$

$4|2w+3| \leq 16$

$|2w+3| \leq 4$

$2w+3 \leq 4$ $\quad 2w+3 \geq -4$

$w \leq \frac{1}{2}$ $\quad w \leq -\frac{7}{2}$

30. $3|5t - 1| + 9 \leq 23$

$-\frac{11}{15} \leq t \leq \frac{17}{15}$

31. $|x + 3| > 9$

$x+3 > 9$ $\quad$ OR $\quad x+3 < -9$

$x > 6$ $\quad$ OR $\quad x < -12$

32. $|x - 5| \geq 8$

$x \leq -3$ or $x \geq 13$

33. $|y - 3| \geq 12$

$y \leq -9$ or $y \geq 15$

34. $|2x + 1| \geq -9$

All Real numbers

(AV must be positive!)

35. $3|2x - 1| \geq 21$

$x \leq -3$ or $x \geq 4$

36. $|3z| - 4 > 8$

$z < -4$ or $z > 4$

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Extra Practice: AV Equations and Inequalities

Solve each equation.

43. $-|4 - 8b| = 12$

$|4-8b| = -12$

NO solution
AV can't be negative

44. $4|3x + 4| = 4x + 8$

$|3x+4| = x+2$

$3x+4 = x+2$ $\quad$ $3x+4 = -x-2$

$2x = -2$ $\quad$ $4x = -6$

$x = -1$ $\quad$ $x = -\frac{3}{2}$

45. $|3x - 1| + 10 = 25$

$|3x-1| = 15$

$3x-1 = 15$ $\quad$ $3x-1 = -15$

$x = \frac{16}{3}$ $\quad$ $x = \frac{-14}{3}$

46. $\frac{1}{2}|3c + 5| = 6c + 4$

$|3c+5| = 12c+8$

$3c+5 = 12c+8$ $\quad$ $3c+5 = -12c-8$

$-3 = 9c$ $\quad$ $15c = -13$

$c = -\frac{1}{3}$ $\quad$ $c = \frac{-13}{15}$ → extraneous

47. $5|6 - 5x| = 15x - 35$

$|6-5x| = 3x-7$

$6-5x = 3x-7$ $\quad$ $6-5x = -3x+7$

$13 = 8x$ $\quad$ $-1 = 2x$

$x = 8/13$ ← Both extraneous → $x = -1/2$

48. $7|8 - 3h| = 21h - 49$

$|8-3h| = 3h-7$

$8-3h = 3h-7$ $\quad$ $8-3h = -3h+7$

$15 = 6h$ $\quad$ $1 = 0$ → no solution

$h = \frac{5}{2}$ → only solution

Solve each inequality. Graph the solutions.

57. $|3x - 4| + 5 \leq 27$

$3x-4 \leq 22$ $\quad$ $3x-4 \geq -22$

$3x \leq 26$ $\quad$ $3x \geq -18$

$x \leq 26/3$ and $x \geq -6$

58. $|2x + 3| - 6 \geq 7$

$2x+3 \geq 13$ or $2x+3 \leq -13$

$x \geq 5$ or $x \leq -8$

63. $\frac{1}{9}|5x - 3| - 3 \geq 2$

$\frac{1}{9}|5x-3| \geq 5$

$|5x-3| \geq 45$

$5x-3 \geq 45$ or $5x-3 \leq -45$

$x \geq \frac{48}{5}$ or $x \leq \frac{-42}{5}$

64. $\frac{1}{11}|2x - 4| + 10 \leq 11$

$\frac{1}{11}|2x-4| \leq 1$

$|2x-4| \leq 11$

$2x-4 \leq 11$ $\quad$ $2x-4 \geq -11$

$x \leq \frac{15}{2}$ and $x \geq -7/2$

65. $\left|\frac{x-3}{2}\right| + 2 < 6$

$\left|\frac{x-3}{2}\right| < 4$

$\frac{x-3}{2} < 4$ and $\frac{x-3}{2} > -4$

$x < 11$ $\quad$ $x > -5$

66. $\left|\frac{x+5}{3}\right| - 3 > 6$

$x < -32$ or $x > 22$