Lesson 1.2 - Domain/Intro to Piecewise Functions

Learning Objectives: SWBAT

- Identify the domain of a function given a set of ordered pairs
- Identify the domain of a function given an equation (state excluded values)
- Evaluate a piecewise function for a given input value

Reflect: In the space below, describe or define the term "Domain" of a function. What is it, what words are associated with it?

Domain (A bit more advanced)

The domain of a function can be described explicitly or it can be *implied* by the expression used to define the function. The **implied domain** is the set of all real numbers for which the expression is defined. For instance, the function

$$f(x) = \frac{1}{x^2 - 4}$$ Domain excludes x-values that result in division by zero.

has an implied domain that consists of all real x other than $x = \pm 2$. These two values are excluded from the domain because division by zero is undefined. Another common type of implied domain is that used to avoid even roots of negative numbers. For example, the function

$$f(x) = \sqrt{x}$$ Domain excludes x-values that result in even roots of negative numbers.

is defined only for $x \geq 0$. So, its implied domain is the interval $[0, \infty)$. In general, the domain of a function *excludes* values that would cause division by zero *or* result in the even root of a negative number.

Exploration

Use a graphing utility to graph $y = \sqrt{4 - x^2}$. What is the domain of this function? Then graph $y = \sqrt{x^2 - 4}$. What is the domain of this function? Do the domains of these two functions overlap? If so, for what values?

Examples: **Finding the the domain of a function**

Find the domain of each function.

a. f: $\{(-3, 0), (-1, 4), (0, 2), (2, 2), (4, -1)\}$

b. $g(x) = -3x^2 + 4x + 5$

c. $h(x) = \dfrac{1}{x + 5}$

d. Volume of a sphere: $V = \frac{4}{3}\pi r^3$

e. $k(x) = \sqrt{4 - 3x}$

Solution

a. The domain of f consists of all first coordinates in the set of ordered pairs.

Domain $= \{-3, -1, 0, 2, 4\}$

b. The domain of g is the set of all *real* numbers.

c. Excluding x-values that yield zero in the denominator, the domain of h is the set of all real numbers x except $x = -5$.

d. Because this function represents the volume of a sphere, the values of the radius r must be positive. So, the domain is the set of all real numbers r such that $r > 0$.

e. This function is defined only for x-values for which $4 - 3x \geq 0$. By solving this inequality, you will find that the domain of k is all real numbers that are less than or equal to $\frac{4}{3}$.

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What is a Piecewise Function?

Library of Parent Functions: Piecewise-Defined Function

A *piecewise-defined function* is a function that is defined by two or more equations over a specified domain. The *absolute value function* given by $f(x) = |x|$ can be written as a piecewise-defined function. The basic characteristics of the absolute value function are summarized below. A review of piecewise-defined functions can be found in the *Study Capsules*.

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

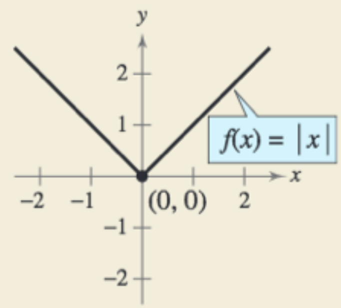
Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Intercept: $(0, 0)$

Decreasing on $(-\infty, 0)$

Increasing on $(0, \infty)$



Example: **Evaluating a Piecewise Function**

Evaluate the function when $x = -1$ and $x = 0$.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution

Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1 = 2.$$

For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = 0 - 1 = -1.$$

Your Turn: Evaluate the following functions for the value(s) given

37. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1)$ (b) $f(0)$ (c) $f(2)$

38. $f(x) = \begin{cases} 2x + 5, & x \leq 0 \\ 2 - x^2, & x > 0 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$

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Practice: Evaluate the following functions for the value(s) given

39. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2)$ (b) $f(1)$ (c) $f(2)$

40. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

(a) $f(-2)$ (b) $f(0)$ (c) $f(1)$

41. $f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2 + 1, & x \ge 2 \end{cases}$

(a) $f(-2)$ (b) $f(1)$ (c) $f(4)$

42. $f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x + 1, & x \ge 1 \end{cases}$

(a) $f(-2)$ (b) $f(\frac{1}{2})$ (c) $f(1)$

In Exercises 47–50, find all real values of x such that $f(x) = 0$.

47. $f(x) = 15 - 3x$ **48.** $f(x) = 5x + 1$ **49.** $f(x) = \dfrac{3x - 4}{5}$ **50.** $f(x) = \dfrac{2x - 3}{7}$

In Exercises 53–62, find the domain of the function.

53. $f(x) = 5x^2 + 2x - 1$ **54.** $g(x) = 1 - 2x^2$ **57.** $f(x) = \sqrt[3]{x - 4}$ **58.** $f(x) = \sqrt[4]{x^2 + 3x}$

59. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$ **60.** $h(x) = \dfrac{10}{x^2 - 2x}$ **61.** $g(y) = \dfrac{y + 2}{\sqrt{y - 10}}$ **62.** $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$