Lesson 1.3 - Difference Quotients

Learning Objectives: SWBAT

- Evaluate a difference quotient

Activity - Exploring difference quotients by evaluating functions

From website - www.purplemath.com/modules/fcnnot2.htm

Part 1 - Evaluate the following (simplify as much as possible)

For $h(w) = w^2 - 3$, find $h(2d + 1)$.

$(2d+1)^2 - 3$

$(4d^2 + 4d + 1) - 3$

$\boxed{4d^2 + 4d - 2}$

Part 2 - Evaluate the following (simplify as much as possible)

Given that $f(x) = 3x^2 + 2x$, find $f(x + h)$.

$3(x+h)^2 + 2(x+h)$

$3(x^2 + 2xh + h^2) + 2x + 2h$

$\boxed{3x^2 + 6xh + 3h^2 + 2x + 2h}$

Part 3 - Evaluate the following (simplify as much as possible)

Given that $f(x) = 3x^2 + 2x$, find $f(x + h) - f(x)$.

$\underbrace{3x^2 + 6xh + 3h^2 + 2x + 2h}_{f(x+h)} - \underbrace{3x^2 + 2x}_{f(x)} = \boxed{6xh + 3h^2 + 2h}$

Part 4 - Evaluate the following (simplify as much as possible)

Given that $f(x) = 3x^2 + 2x$, find $\dfrac{f(x+h) - f(x)}{h}$ $= \dfrac{6xh + 3h^2 + 2h}{h} = \dfrac{h(6x + 3h + 2)}{h}$

$= \boxed{6x + 3h + 2}$

$h \neq 0$

(This type of functional expression is called a "difference quotient", and is actually something you will see again in calculus. I guess the reason this sort of exercise crops up so commonly in algebra is that they're trying to "prep" you.

(But, to be fair, it's not like anybody remembers these by the time they get to calculus, so it's really a lot of work for no real purpose, in my opinion. However, this type of problem is quite popular, so you should expect to need to know how to do it, and should expect to see one on the next test.)

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Example: **Evaluating a difference quotient**

For $f(x) = x^2 - 4x + 7$, find $\dfrac{f(x+h) - f(x)}{h}$.

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{[(x+h)^2 - 4(x+h) + 7] - (x^2 - 4x + 7)}{h}$$

$$= \frac{x^2 + 2xh + h^2 - 4x - 4h + 7 - x^2 + 4x - 7}{h}$$

$$= \frac{2xh + h^2 - 4h}{h}$$

$$= \frac{h(2x + h - 4)}{h} = 2x + h - 4, \quad h \neq 0$$

Your Turn: Evaluate the Difference quotient and simplify your answer

87. $f(x) = 2x$, $\dfrac{f(x+c) - f(x)}{c}$, $c \neq 0$

$= \dfrac{2(x+c) - 2x}{c}$

$= \dfrac{2x + 2c - 2x}{c} = \dfrac{2c}{c} = \boxed{2}$

88. $g(x) = 3x - 1$, $\dfrac{g(x+h) - g(x)}{h}$, $h \neq 0$

$= \dfrac{3(x-h) - 1 - (3x-1)}{h}$

$= \dfrac{3x + 3h - 1 - 3x + 1}{h} = \dfrac{+3h}{h} = \boxed{+3}$

89. $f(x) = x^2 - x + 1$, $\dfrac{f(2+h) - f(2)}{h}$, $h \neq 0$

$= \dfrac{(h+2)^2 - (h+2) + 1 - ((2)^2 - (2) + 1)}{h}$

$= \dfrac{h^2 + 4h + 4 - h - 2 + 1 - (4 - 2 + 1)}{h}$

$= \dfrac{h^2 + 3h + 4 - 2 - 4 + 2 - 1}{h}$

$= \dfrac{h^2 + 3h}{h} = \dfrac{h(3+h)}{h} = \boxed{3+h}$

90. $f(x) = x^3 + x$, $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$

$\dfrac{(x+h)^3 + (x+h) - (x^3 + x)}{h}$

$\boxed{3x^2 + 3hx + h^2 + 1}$

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Practice

1. Given $f(x) = 2x^2 - x$, find the following and simplify.

(a). $f(x+h)$ (b). $f(x+h) - f(x)$ (c). $\dfrac{f(x+h) - f(x)}{h}$

$2(x+h)^2 - (x+h)$

$2(x^2 + xh + h^2) - x - h$

$2x^2 + 2xh + 2h^2 - x - h$

$2x^2 + 2xh + 2h^2 - x - h - (2x^2 - x)$

$2xh + 2h^2 - h$

$\dfrac{2xh + 2h^2 - h}{h}$

$= 2x + 2h - 1$

$4x + 2h - 1$

2. Given $f(x) = 1 - x^2$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$.

$-2x - h$

3. Given $C(x) = 2x^2 - 4x + 3$, find and simplify $\dfrac{C(x+h) - C(x)}{h}$

$4x + 2h - 4$

4. Given $p(q) = q^2 + 2q - 5$, find and simplify $\dfrac{p(q+h) - p(q)}{h}$

$2q + h + 2$