Lesson 1.6 - Graphing Piecewise Functions

Learning Objectives: SWBAT

- Graph a piece wise function by hand
- Write the equation of a piecewise function

Example: Sketch the graph of $f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$ by hand.

Solution

This piecewise-defined function is composed of two linear functions. At and to the left of $x = 1$, the graph is the line given by $y = 2x + 3$. To the right of $x = 1$, the graph is the line given by $y = -x + 4$ (see Figure 1.33). Notice that the point $(1, 5)$ is a solid dot and the point $(1, 3)$ is an open dot. This is because $f(1) = 5$.

- *Steps*

1. Draw dotted line to set boundary of interval(s) for each "piece" of graph
2. Evaluate the function by the boundary value (1 for this example)
3. Your starting point to graph is the boundary value (x) and the result of the evaluation in step 3 (y)
4. Determine if the point is "open or closed" and graph the point
5. Use the slope of the equation to graph the remainder of the line. Ususally, you will be going in the negative direction. All you need is the boundary point and one other to graph the line
6. Repeat the steps for the second function (if there are 3 functions you will have 3 "pieces of graph"

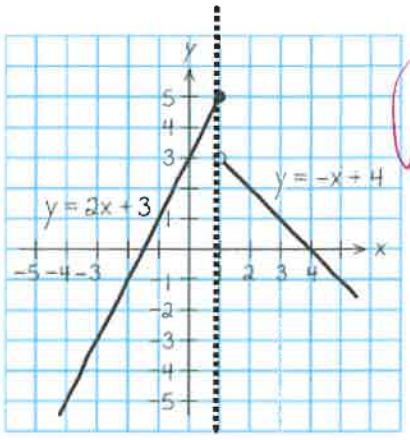


Figure 1.33

Your Turn

- Graph the following function by hand

$$f(x) = \begin{cases} -2x - 4 & \text{if } x \leq 2 \\ 4x - 9 & \text{if } x > 2 \end{cases}$$

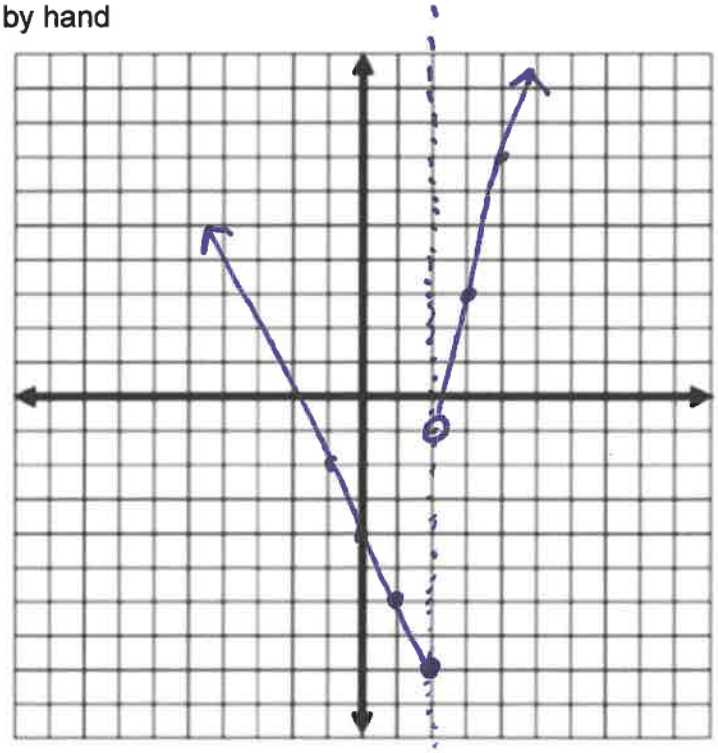
Which Interval = 2 boundary

$f(2) = -2(2) - 4 = -8 \rightarrow (2, -8)$ closed

$f(1) = -2(1) - 4 = -6 \rightarrow (1, -6)$

$f(2) = 4(2) - 9 = -1 \rightarrow (2, -1)$ open

$f(3) = 4(3) - 9 = 3 \rightarrow (3, 3)$



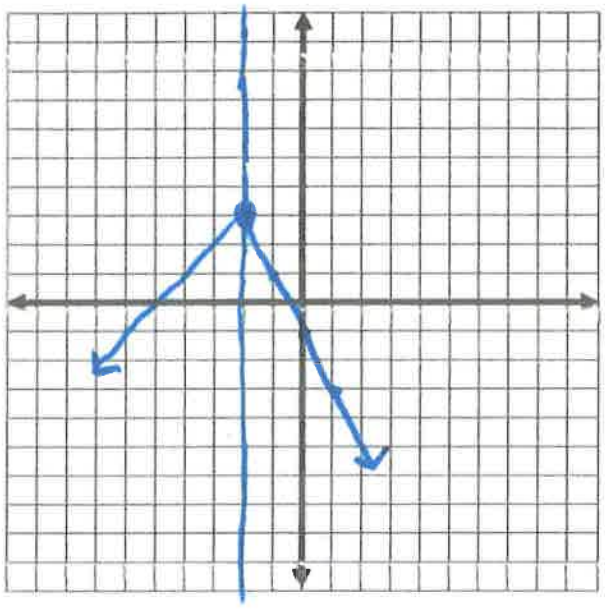
$x = 2$ boundary

you can also use slope to find points on either side of the boundary when the pieces are linear

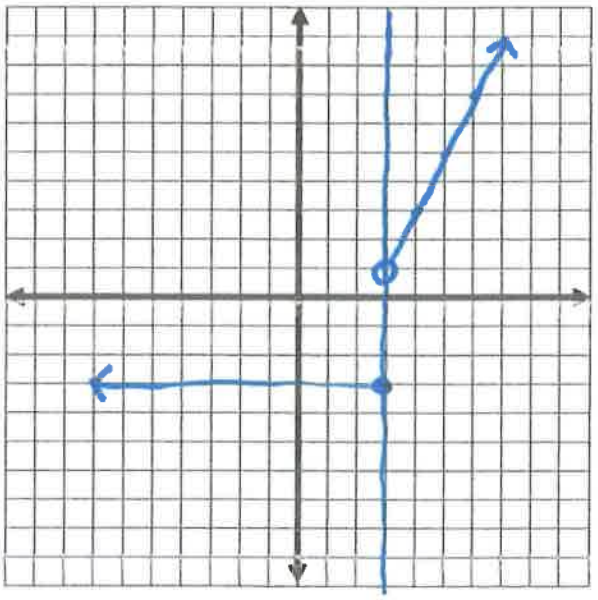
Lesson 1.6 Practice

Piecewise Functions Practice

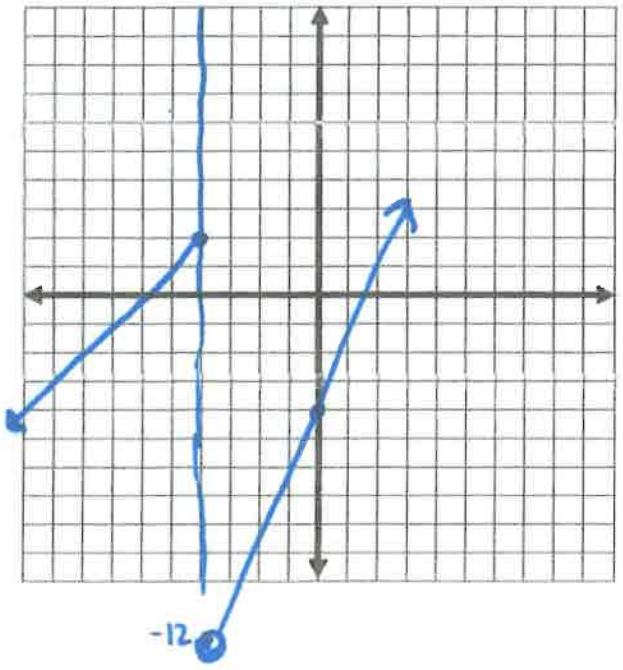
1. $f(x)=\begin{cases} x+5 & x<-2 \\ -2x-1 & x\geq -2 \end{cases}$



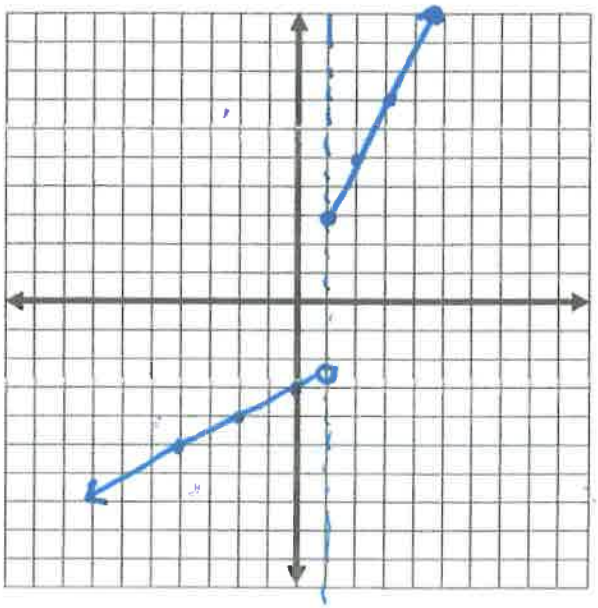
2. $f(x)=\begin{cases} -3 & x\leq 3 \\ 2x-5 & x>3 \end{cases}$



3. $f(x)=\begin{cases} x+6, & x\leq -4 \\ 2x-4, & x>-4 \end{cases}$



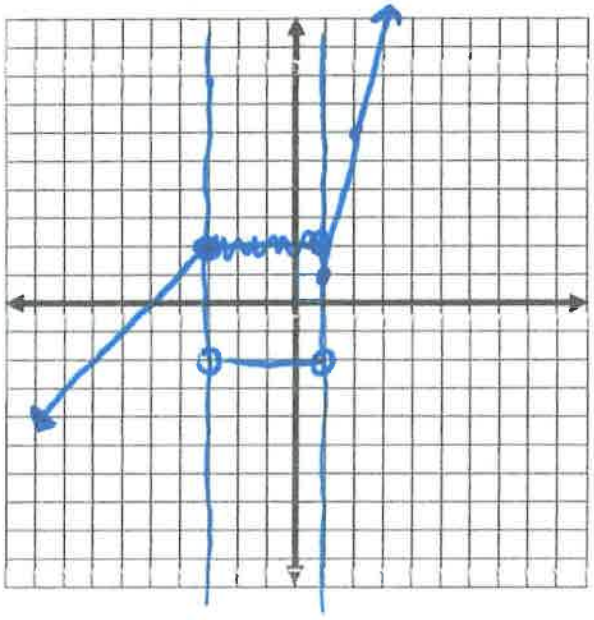
4. $f(x)=\begin{cases} 2x+1 & x\geq 1 \\ \frac{x}{2}-3 & x<1 \end{cases}$



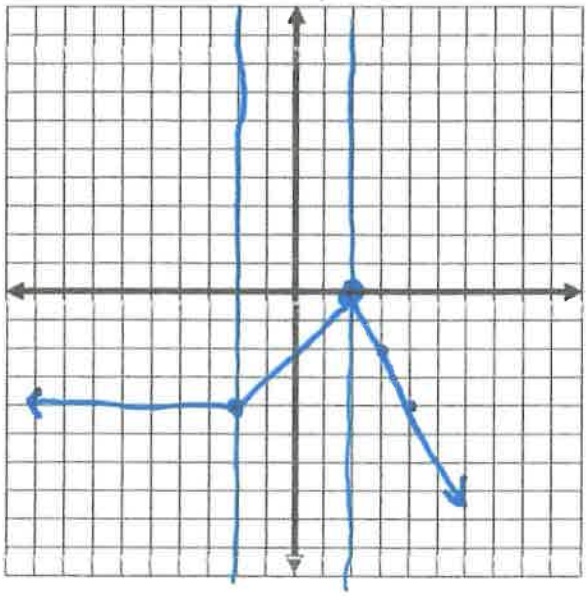
Lesson 1.6 Practice

Piecewise Functions Practice

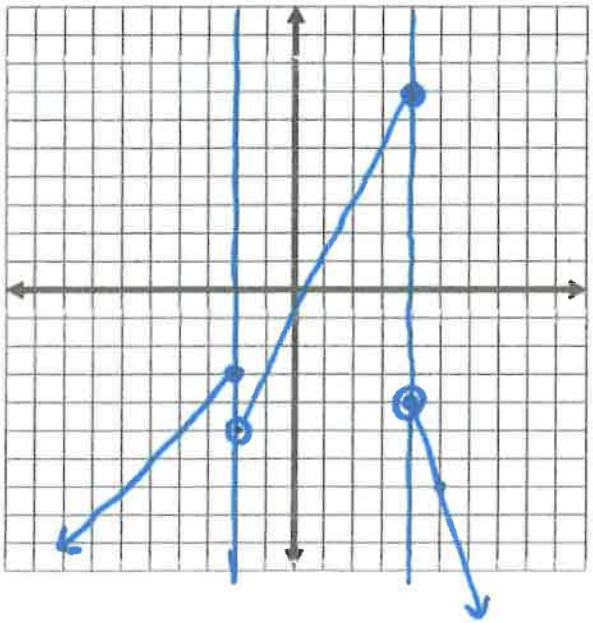
5. $g(x) = \begin{cases} x+5, & x \le -3 \\ -2, & -3 < x < 1 \\ 5x-4, & x \ge 1 \end{cases}$



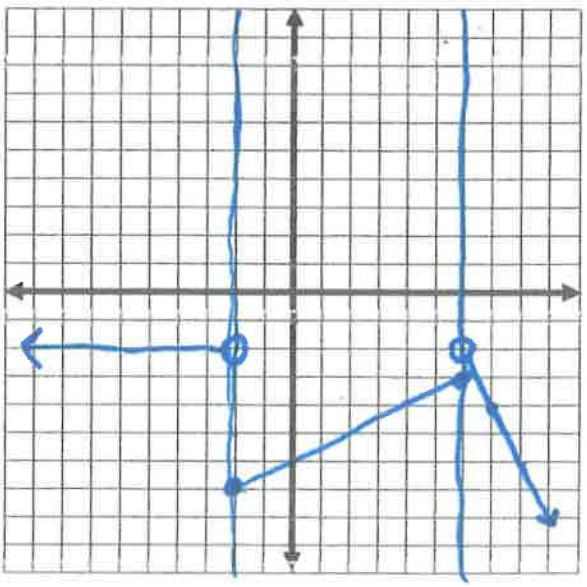
6. $f(x) = \begin{cases} -4, & x \le -2 \\ x-2, & -2 < x < 2 \\ -2x+4, & x \ge 2 \end{cases}$



7. $f(x) = \begin{cases} x-1 & \text{if} \quad x \le -2 \\ 2x-1 & \text{if} \quad -2 < x \le 4 \\ -3x+8 & \text{if} \quad x > 4 \end{cases}$



8. $f(x) = \begin{cases} 5 & \text{if} \quad x < -2 \\ \frac{1}{2}x-6 & \text{if} \quad -2 \le x \le 6 \\ -2x+10 & \text{if} \quad x > 6 \end{cases}$



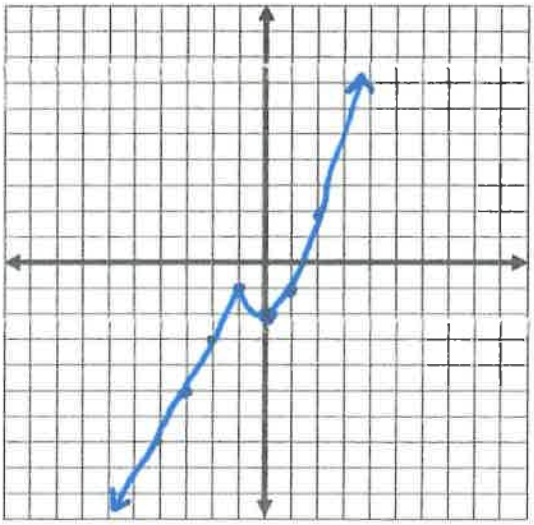
Lesson 1.6 Practice

Piecewise Functions Day 2 Examples

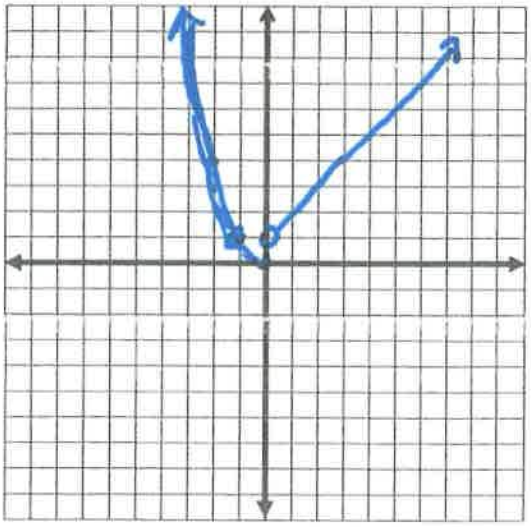
2x+1
f(-1) = -1
f(-2) = -3

x²-2
f(-1) = -1
f(0) = -2

1. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$

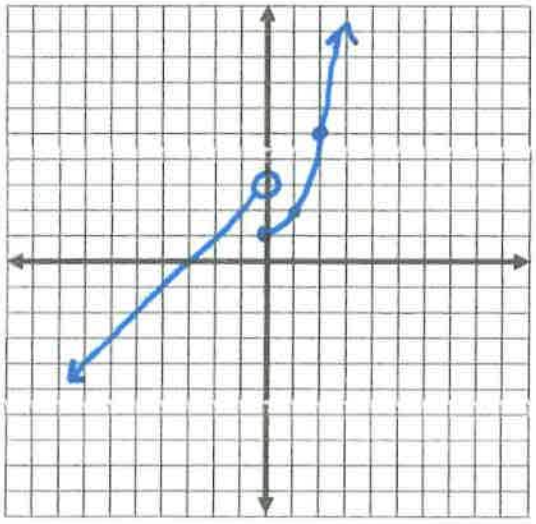


2. $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$

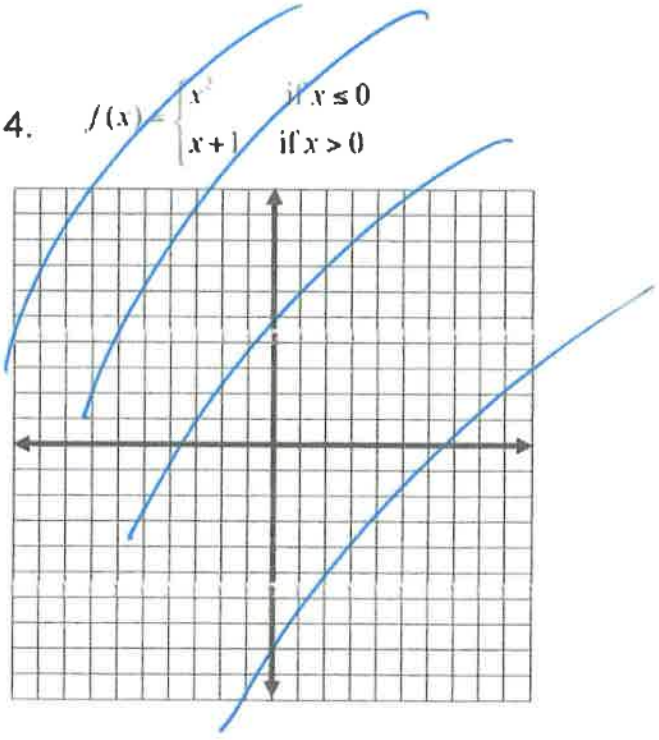


x²+1
f(0) = 1
f(1) = 2

3. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$



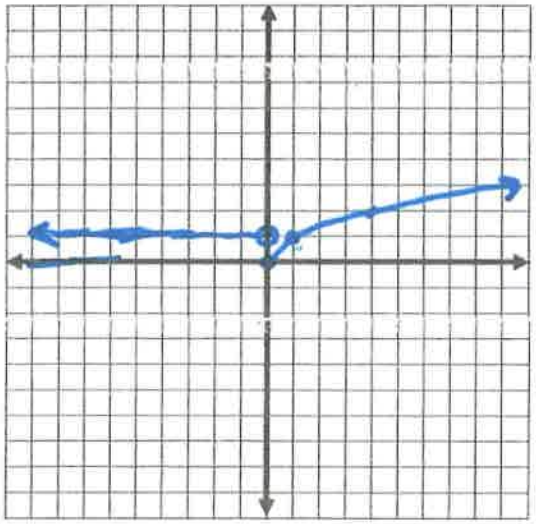
4. $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x + 1 & \text{if } x > 0 \end{cases}$



Lesson 1.6 Practice

Piecewise Functions Day 2 Examples

5. $f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{x}, & x \geq 0 \end{cases}$



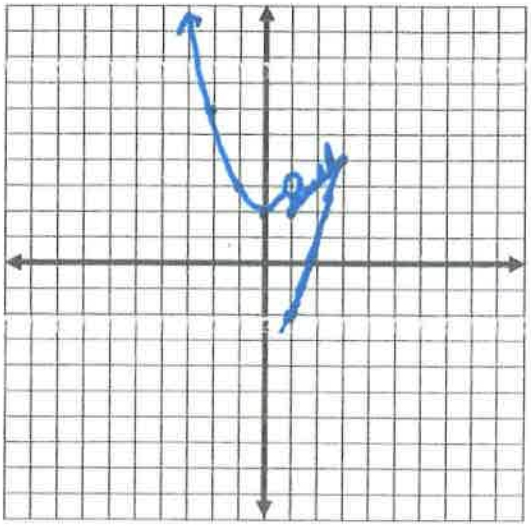
$\sqrt{x}$

$f(0) = 0$

$f(1) = 1$

$f(4) = 2$

6. $f(x) = \begin{cases} x^2 + 2 & \text{if } x < 1 \\ 3x - 5 & \text{if } 1 \le x \le 3 \end{cases}$



$x^2 + 2$

$f(1) = 3$

$f(0) = 2$

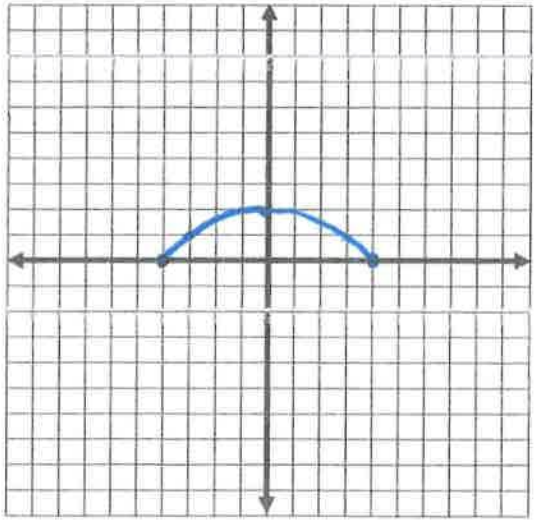
$f(-1) = 3$

$3x - 5$

$f(1) = -2$

$f(3) = 4$

7. $f(x) = \begin{cases} \sqrt{4 + x}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



$\sqrt{4+x}$

$f(0) = 2$

$f(-3) = 1$

$f(-4) = 0$

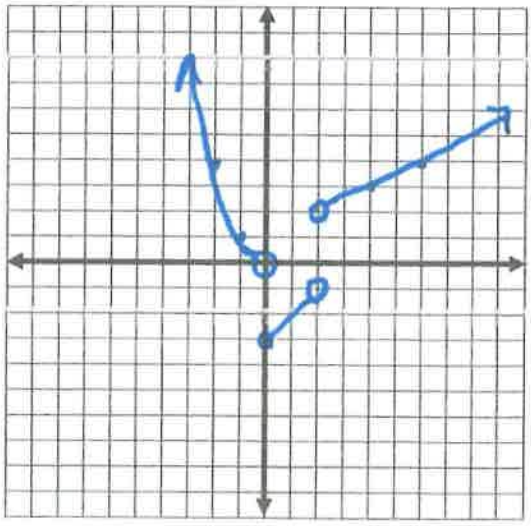
$\sqrt{4-x}$

$f(0) = 2$

$f(3) = 1$

$f(4) = 0$

8. $f(x) = \begin{cases} x & \text{if } x < 0 \\ x - 3 & \text{if } 0 \le x < 2 \\ \frac{1}{2}x + 1 & \text{if } x > 2 \end{cases}$



Piecewise Functions Connecting Equations to Graphs

Match the piecewise function with its graph.

(E) 13. $f(x) = \begin{cases} x - 4, & \text{if } x \le 1 \\ 3x, & \text{if } x > 1 \end{cases}$

(B) 14. $f(x) = \begin{cases} x + 4, & \text{if } x \le 0 \\ 2x + 4, & \text{if } x > 0 \end{cases}$

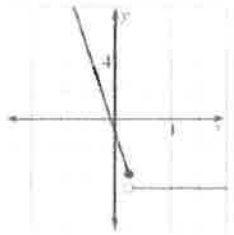
(D) 15. $f(x) = \begin{cases} 3x - 2, & \text{if } x \le 1 \\ x + 2, & \text{if } x > 1 \end{cases}$

(F) 16. $f(x) = \begin{cases} 2x + 3, & \text{if } x \ge 0 \\ x + 4, & \text{if } x < 0 \end{cases}$

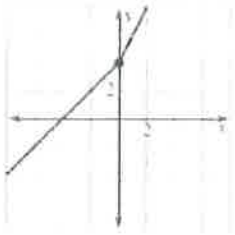
(C) 17. $f(x) = \begin{cases} 3x - 1, & \text{if } x \ge -1 \\ -5, & \text{if } x < -1 \end{cases}$

(A) 18. $f(x) = \begin{cases} -3x - 1, & \text{if } x \le 1 \\ -5, & \text{if } x > 1 \end{cases}$

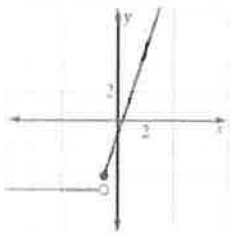
A.



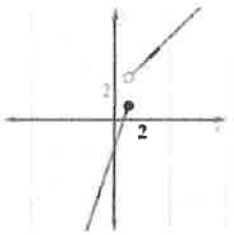
B.



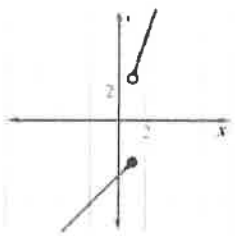
C.



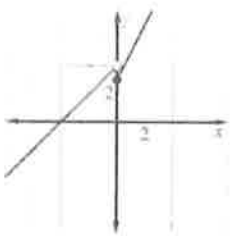
D.



E.



F.



Match the piecewise functions to their graphs.

(B) 1. $f(x) = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}$

(A) 2. $f(x) = \begin{cases} -3, & x \le 1 \\ 2x + 1, & x > 1 \end{cases}$

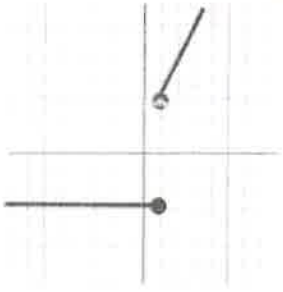
(D) 3. $f(x) = \begin{cases} -x - 3, & x < 0 \\ 4, & x \ge 2 \end{cases}$

(E) 4. $f(x) = \begin{cases} -1, & x < 2 \\ x^2, & 2 \le x \le 2 \\ x, & x > 2 \end{cases}$

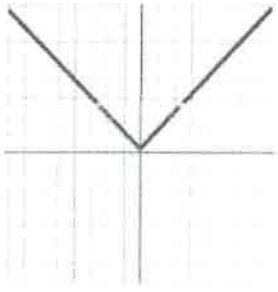
(F) 5. $f(x) = \begin{cases} 3x - 1, & x \le \\ -\frac{1}{2}x - 1, & x > \end{cases}$

(C) 6. $f(x) = \begin{cases} 5, & x < -3 \\ x - 3, & x \ge -3 \end{cases}$

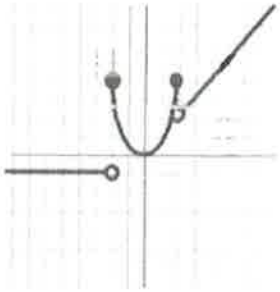
A



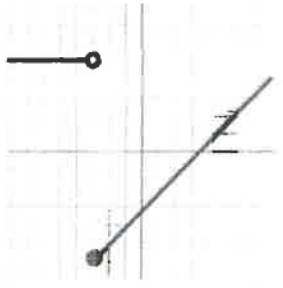
B



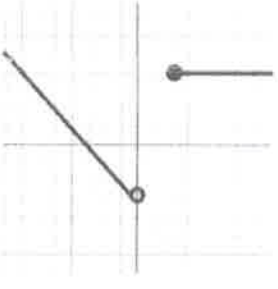
E



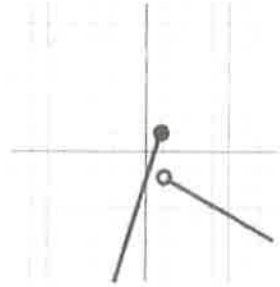
C



D



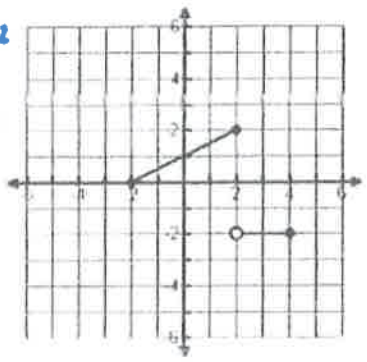
F



Piecewise Functions Writing Equations

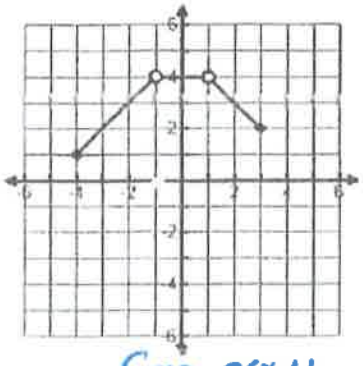
Write the equation for each piecewise function graph below

1)



$f(x) = \begin{cases} \frac{1}{2}x+1, & -2 \le x \le 2 \\ -2, & 2 < x \le 4 \end{cases}$

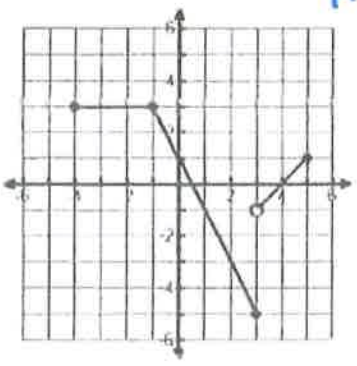
2)



$f(x) = \begin{cases} x+5, & -5 \le x < -1 \\ 4, & -1 < x < 1 \\ -x+3, & x > 1 \end{cases}$

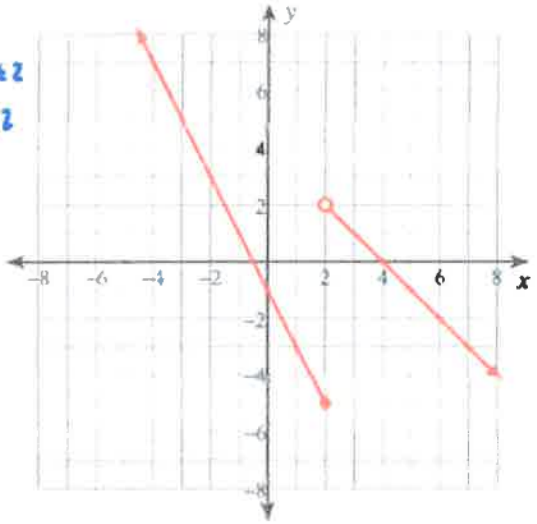
$1 < x \le 3$

3)

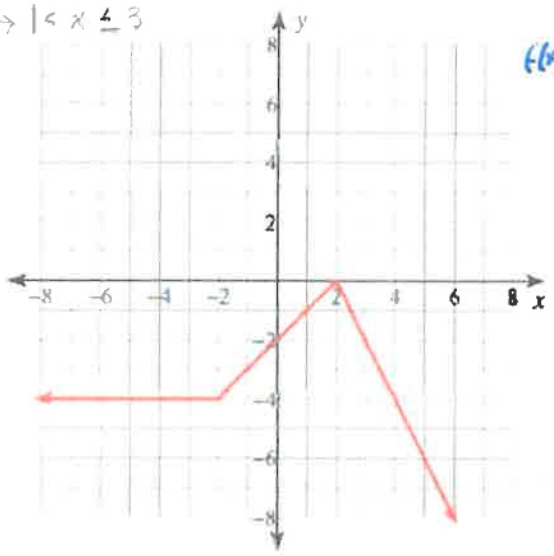


$f(x) = \begin{cases} 3, & -5 \le x \le 1 \\ -2x+1, & -1 < x \le 3 \\ x-4, & x > 3 \end{cases}$

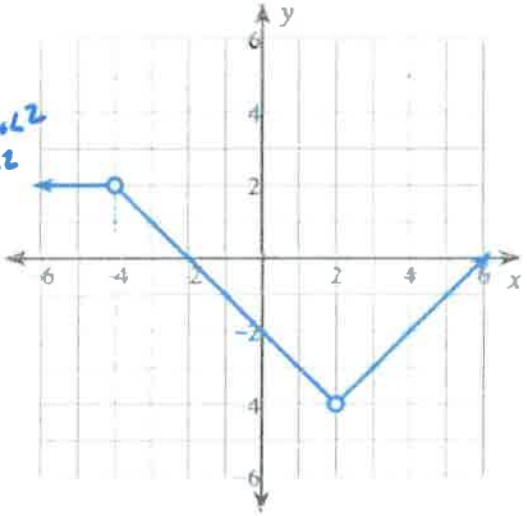
$3 < x \le 5$



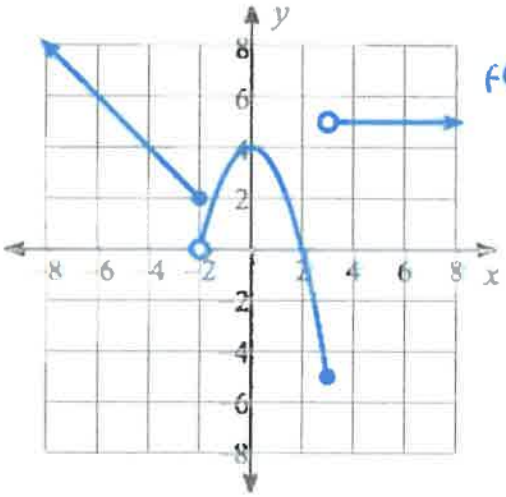
$f(x) = \begin{cases} -2x-1, & x \le 2 \\ -x+4, & x > 2 \end{cases}$



$f(x) = \begin{cases} -4, & x \le -2 \\ x-2, & -2 < x \le 2 \\ -2x+4, & x > 2 \end{cases}$



$f(x) = \begin{cases} 2, & x < -4 \\ -x-2, & -4 < x < 2 \\ x-6, & x > 2 \end{cases}$



$f(x) = \begin{cases} -x, & x \le -2 \\ -x^2+4, & -2 < x \le 3 \\ 5, & x > 3 \end{cases}$