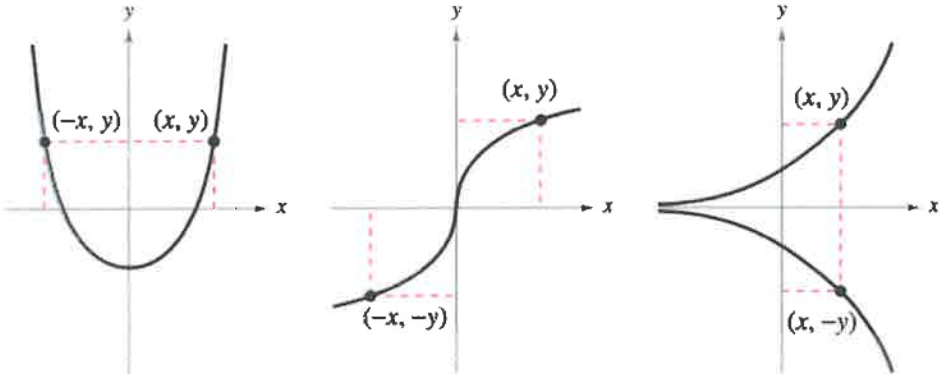
Lesson 1.7 - Even and Odd Functions

Learning Objectives: SWBAT

- Explain the difference (graphically) between and even and odd function
- Determine whether or not a given equation is even, odd or neither

A graphic look at Even and Odd Functions:

A graph has *symmetry with respect to the y-axis* if whenever (x, y) is on the graph, so is the point $(-x, y)$. A graph has *symmetry with respect to the origin* if whenever (x, y) is on the graph, so is the point $(-x, -y)$. A graph has *symmetry with respect to the x-axis* if whenever (x, y) is on the graph, so is the point $(x, -y)$. A function whose graph is symmetric with respect to the y-axis is an **even function.** A function whose graph is symmetric with respect to the origin is an **odd function.** A graph that is symmetric with respect to the x-axis is not the graph of a function (except for the graph of $y = 0$). These three types of symmetry are illustrated in Figure 1.34.



Symmetric to y-axis ***Even function***

Symmetric to origin ***Odd function***

Symmetric to x-axis ***Not a function***

Figure 1.34

Test for Even and Odd Functions

A function f is **even** if, for each x in the domain of f, $f(-x) = f(x)$.
A function f is **odd** if, for each x in the domain of f, $f(-x) = -f(x)$.

Examples: Determine whether each function is even, odd, or neither.

a. $g(x) = x^3 - x$
b. $h(x) = x^2 + 1$
c. $f(x) = x^3 - 1$

Plug (-x) in for x

a. This function is odd because

$$\begin{aligned} g(-x) &= (-x)^3 - (-x) \\ &= -x^3 + x \\ &= -(x^3 - x) \\ &= -g(x). \end{aligned}$$

b. This function is even because

$$\begin{aligned} h(-x) &= (-x)^2 + 1 \\ &= x^2 + 1 \\ &= h(x). \end{aligned}$$

c. Substituting $-x$ for x produces

$$\begin{aligned} f(-x) &= (-x)^3 - 1 \\ &= -x^3 - 1. \end{aligned}$$

Because $f(x) = x^3 - 1$ and $-f(x) = -x^3 + 1$, you can conclude that $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$. So, the function is neither even nor odd.

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Practice: Determine if the following functions are even, odd or neither. Graph each function on Desmos to verify your answer

59. $f(t) = t^2 + 2t - 3$

$f(-t) = (-t)^2 + 2(-t) - 3$

$= t^2 - 2t - 3$

Neither

60. $f(x) = x^6 - 2x^2 + 3$

$f(-x) = (-x)^6 - 2(-x)^2 + 3$

$= x^6 - 2x^2 + 3$

EVEN

61. $g(x) = x^3 - 5x$

$f(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -(x^3 - 5x)$

odd

62. $h(x) = x^3 - 5$

$f(-x) = (-x)^3 - 5$

$h = -x^3 - 5$

Neither

63. $f(x) = x\sqrt{1 - x^2}$

$f(-x) = (-x)\sqrt{1 - (-x)^2}$

$= -x\sqrt{1 - x^2}$

odd

64. $f(x) = x\sqrt{x + 5}$

$g(-x) = (-x)\sqrt{(-x) + 5}$

$= -x\sqrt{-x + 5}$

Neither

65. $g(s) = 4s^{2/3}$

$g(-s) = 4\sqrt[3]{(-s)^2}$

$= 4\sqrt[3]{s^2}$

EVEN

66. $f(s) = 4s^{3/2}$

$f(-s) = 4\sqrt{(-s)^3}$

$= 4\sqrt{-s^3}$

Neither

Find the coordinates of a second point on the graph given the first point and the function is even or odd (each question will have two answers)

67. $\left(-\frac{3}{2}, 4\right)$

if even: $\left(\frac{3}{2}, 4\right)$

if odd: $\left(\frac{3}{2}, -4\right)$

68. $\left(-\frac{5}{3}, -7\right)$

if even: $\left(\frac{5}{3}, -7\right)$

if odd: $\left(\frac{5}{3}, 7\right)$

69. $(4, 9)$

if even $(-4, 9)$

if odd $(-4, -9)$

70. $(5, -1)$

if even $(-5, -1)$

if odd $(-5, 1)$

71. $(x, -y)$

if even: $(-x, -y)$

if odd: $(-x, y)$

72. $(2a, 2c)$

if even $(-2a, 2c)$

if odd $(-2a, -2c)$

Practice: Determine if the following functions are even, odd or neither. Graph each function on Desmos to verify your answer

73. $f(x) = 5$

Even

74. $f(x) = -9$

Even

75. $f(x) = 3x - 2$

Neither

76. $f(x) = 5 - 3x$

Neither

77. $h(x) = x^2 - 4$

even

78. $f(x) = -x^2 - 8$

even