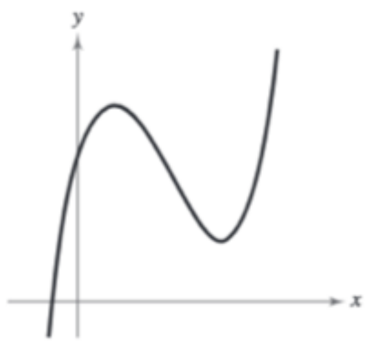
Lesson 1.8 - Transformations & Polynomial End Behavior

Learning Objectives: SWBAT

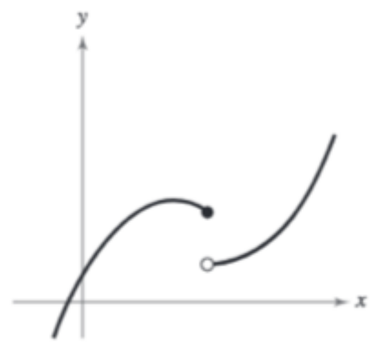
- Use transformations to sketch the graphs of polynomial functions
- Use the leading coefficient test to determine the end behavior of polynomial function graphs

You should be able to sketch accurate graphs of polynomial functions of degrees 0, 1, and 2. The graphs of polynomial functions of degree greater than 2 are more difficult to sketch by hand. However, in this section you will learn how to recognize some of the basic features of the graphs of polynomial functions. Using these features along with point plotting, intercepts, and symmetry, you should be able to make reasonably accurate sketches *by hand*.

The graph of a polynomial function is **continuous.** Essentially, this means that the graph of a polynomial function has no breaks, holes, or gaps, as shown in Figure 2.14. Informally, you can say that a function is continuous if its graph can be drawn with a pencil without lifting the pencil from the paper.



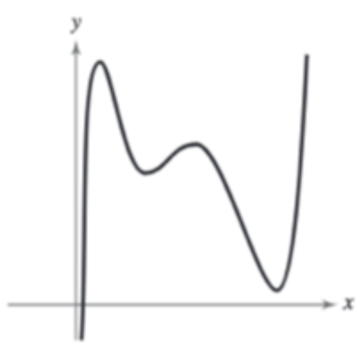
(a) Polynomial functions have continuous graphs.



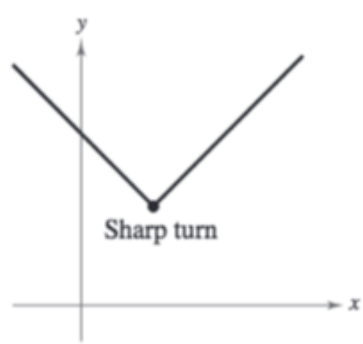
(b) Functions with graphs that are not continuous are not polynomial functions.

Figure 2.14

Another feature of the graph of a polynomial function is that it has only smooth, rounded turns, as shown in Figure 2.15(a). It cannot have a sharp turn such as the one shown in Figure 2.15(b).



(a) Polynomial functions have graphs with smooth, rounded turns.



(b) Functions with graphs that have sharp turns are not polynomial functions.

Figure 2.15

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Examples: Using Transformations to determine the general shape of the graph:

Sketch the graphs of (a) $f(x) = -x^5$, (b) $g(x) = x^4 + 1$, and (c) $h(x) = (x + 1)^4$.

Solution

a. Because the degree of $f(x) = -x^5$ is odd, the graph is similar to the graph of $y = x^3$. Moreover, the negative coefficient reflects the graph in the x-axis, as shown in Figure 2.16.

b. The graph of $g(x) = x^4 + 1$ is an upward shift of one unit of the graph of $y = x^4$, as shown in Figure 2.17.

c. The graph of $h(x) = (x + 1)^4$ is a left shift of one unit of the graph of $y = x^4$, as shown in Figure 2.18.

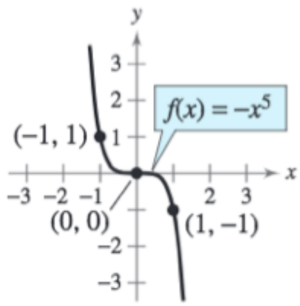

Figure 2.16

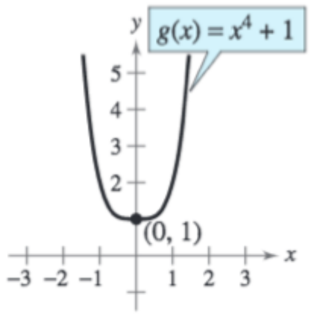

Figure 2.17

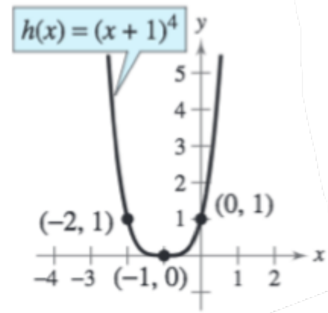

Figure 2.18

Things to remember about transformations:

- Vertical transformations happen "outside the function"
- Horizontal transformations happen "inside the function"
- Any transformation "inside the function" occurs using opposite logic. For example, in problem "c" above, the function $(x + 1)^4$, the horizontal translation goes LEFT where you would normally think it goes right because of the + sign
- Any polynomial with an even degree will have end behavior that points in the same direction
- Any polynomial with an odd degree will have end behavior that points in opposite directions.

More about End Behavior

- The **lead coefficient test** allows us to determine the end behavior of a polynomial.
- If the lead coefficient is > 0, then the graph's end behavior is the same as the parent function's end behavior
  - > For even degree functions the end behavior of the function with rise right and rise left
  - > For odd degree functions, the end behavior will fall left, rise right
- **If the lead coefficient is < 0**, then the graph is reflected and the end behavior is inverted
  - > For even degree functions the end behavior of the function with fall right and fall left
  - > For odd degree functions, the end behavior will rise left, fall right

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Examples: Applying the lead coefficient test

Use the Leading Coefficient Test to describe the right-hand and left-hand behavior of the graph of each polynomial function.

a. $f(x) = -x^3 + 4x$ **b.** $f(x) = x^4 - 5x^2 + 4$ **c.** $f(x) = x^5 - x$

Solution

a. Because the degree is odd and the leading coefficient is negative, the graph rises to the left and falls to the right, as shown in Figure 2.19.

b. Because the degree is even and the leading coefficient is positive, the graph rises to the left and right, as shown in Figure 2.20.

c. Because the degree is odd and the leading coefficient is positive, the graph falls to the left and rises to the right, as shown in Figure 2.21.

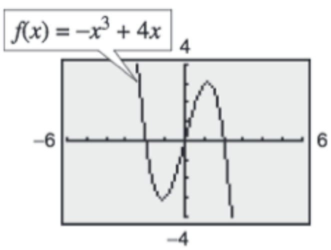


Figure 2.19

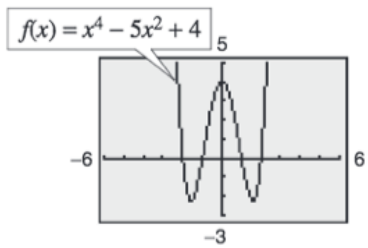


Figure 2.20

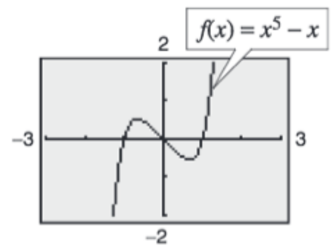
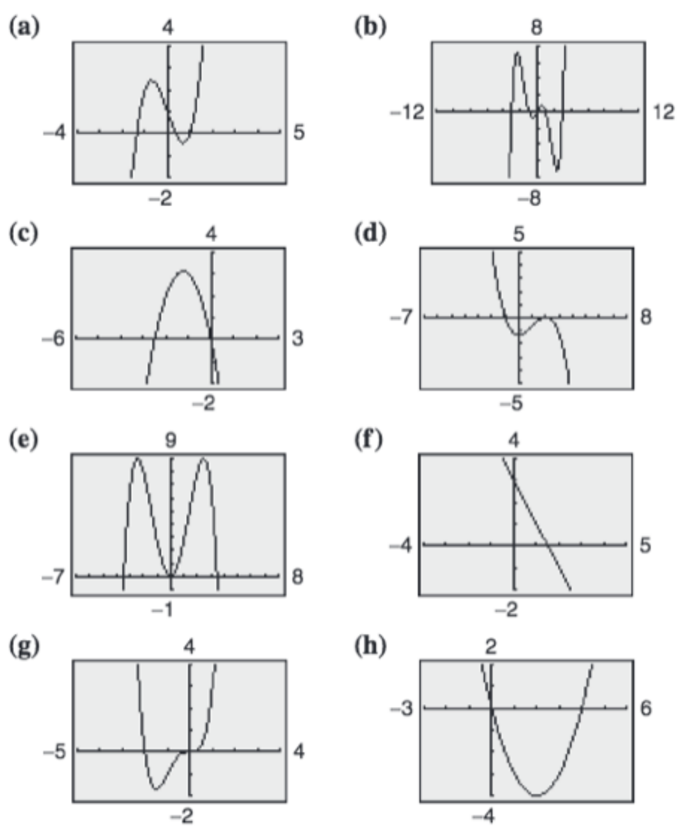


Figure 2.21

Practice **In Exercises 1–8, match the polynomial function with its graph. [The graphs are labeled (a) through (h).]**



1. $f(x) = -2x + 3$

2. $f(x) = x^2 - 4x$

3. $f(x) = -2x^2 - 5x$

4. $f(x) = 2x^3 - 3x + 1$

5. $f(x) = -\frac{1}{4}x^4 + 3x^2$

6. $f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3}$

7. $f(x) = x^4 + 2x^3$

8. $f(x) = \frac{1}{5}x^5 - 2x^3 + \frac{9}{5}x$

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Practice For each equation below, describe the transformations present from the parent function (Reflections, horizontal and vertical translations only)

9. $y = x^3$

(a) $f(x) = (x - 2)^3$ (b) $f(x) = x^3 - 2$

(c) $f(x) = -\frac{1}{2}x^3$ (d) $f(x) = (x - 2)^3 - 2$

10. $y = x^4$

(a) $f(x) = (x + 5)^4$ (b) $f(x) = x^4 - 5$

(c) $f(x) = 4 - x^4$ (d) $f(x) = \frac{1}{2}(x - 1)^4$

In Exercises 15–22, use the Leading Coefficient Test to describe the right-hand and left-hand behavior of the graph of the polynomial function. Use a graphing utility to verify your result.

15. $f(x) = 2x^4 - 3x + 1$

16. $h(x) = 1 - x^6$

17. $g(x) = 5 - \frac{7}{2}x - 3x^2$

18. $f(x) = \frac{1}{3}x^3 + 5x$

19. $f(x) = \dfrac{6x^5 - 2x^4 + 4x^2 - 5x}{3}$

20. $f(x) = \dfrac{3x^7 - 2x^5 + 5x^3 + 6x^2}{4}$

21. $h(t) = -\frac{2}{3}(t^2 - 5t + 3)$