Lesson 2.10 Answers

Your Turn 1 - Population Growth

a. decreasing because exponent is negative

b. 1990: $t=0$, $P=372{,}550$ people

2000: $t=10$, $P=335{,}349$ people

2004: $t=14$, $P=321{,}530$ people

c. $300 = 372.55e^{-.01052t}$

$$\frac{300}{372.55} = e^{-.01052t}$$

$$\ln\left(\frac{300}{372.55}\right) = -.01052t$$

$$\frac{\ln\left(\frac{300}{372.55}\right)}{-.01052}$$

$\boxed{20.6\text{ yrs} = t} \rightarrow \boxed{\text{during year } 2010}$

Your Turn 2 - Richter Scale

a. $I = 10^{6.1} = 1{,}258{,}925$

b. $I = 10^{7.6} = 39{,}810{,}717$

c. $I = 10^{9} =$ 1 billion

Your Turn 3 - Compound Interest

$12{,}400 = 8000\left(1+\frac{.07}{4}\right)^{4t}$ → divide by 8000 and add info in ( )

$1.55 = (1.0175)^{4t}$ → turn exponent into multiplication after taking log of both sides

$\frac{\log 1.55}{\log 1.0175} = 4t(\log 1.0175)$ → divide both sides by log 1.0175

$25.2616 = 4t$ → divide by 4

$\boxed{6.3\text{ years} = t}$

Lesson 2.10 Answers

Practice Population 1

a. $180 = 134e^{10k}$

$\frac{180}{134} = e^{10k}$

$\ln\left(\frac{180}{134}\right) = 10k$

$\boxed{k = .0295}$

b. $t = 20$

$P = 134e^{(.0295)(20}$

$\boxed{P = 241,734 \text{ people}}$

Practice Population 2

a $478 = 258e^{10k}$

$\frac{478}{258} = e^{10k}$

$\ln\left(\frac{478}{258}\right) = 10k$

$\boxed{k = .0617}$

b. $t = 20$

$P = 258e^{(.0167)(20)}$

$\boxed{P = 886,215 \text{ people}}$

Practice Richterscale

$(\underset{\uparrow}{R})$ magnitude $= \log I$

a: $R = \log(39,811,000) = 7.6$ on Richterscale

b: $R = \log(12,589,000) = 7.1$ on Richterscale

c: $R = \log(251,200) = 5.4$ on Richterscale

Lesson 2.10 Answers

Practice Compound Interest → use $\boxed{Pe^{rt} = FV}$

Table	Initial Investment	Annual Interest Rate	Time to Double	Amount after 10 yrs
7.	\$10,000	3.5%	19.8 yrs	\$14,190.68
8.	\$2,000	1.5%	46.2 yrs	\$2,323.67
9.	\$7,500	3.3%	21 years	\$10,432.26
10.	\$1,000	5.8%	12 years	\$1,786.04
11.	\$5,000	1.25%	55.5 years	\$5,665.74
12.	\$300	2.5%	27.7 yrs	\$385.21
13.	\$63,762.82	4.5%	15.4 yrs	\$100,000.00
14.	2046.83	2%	34.7 yrs	\$2,500.00

1. FV = 7,396.29
2. t = 10.2 years
3. t = 5.4 years
4. P = \$12,738.85
5. t = 3.1 years
6. t = 7.74 years

Lesson 2.10 Exponential and Logarithmic Applications

Exponential Growth / Decay Model – Given exponential growth or decay the amount P after time t is given by the following formula: $P = P_0 e^{kt}$

Here P_0 is the initial amount and k is the exponential growth/decay rate.

If k is positive then we will have a growth model and if k is negative then we will have a decay model.

Use the exponential growth/decay model to answer the questions.

A. A certain bacterium has an exponential growth rate of 25% per day. If we start with 0.5 gram and provide unlimited resources how much bacteria can we grow in 2 weeks?

Given $P_0 = .5$ gm

$k = 25\% = .25$

$t = 14$ days

$P = P_0 e^{kt}$

$= .5e^{.25(14)}$

$= .5e^{3.5} \approx 16.56$

We can grow 16.56 gms in 2 weeks.

B. During its exponential growth phase, a certain bacterium can grow from 5,000 cells to 12,000 cells in 10 hours. At this rate how long will it take to grow to 50,000 cells?

Given $P_0 = 5{,}000$

① find k (12,000 cells in 10 hrs)

$P = P_0 e^{kt}$

$12{,}000 = 5{,}000 e^{k \cdot 10}$

$\frac{12{,}000}{5{,}000} = e^{10k}$

$\ln(12/5) = 10k \cdot \ln e$

$k = \frac{\ln(2.4)}{10}$

② Model:

$P = 5{,}000 e^{\left(\frac{\ln(2.4)}{10} \cdot t\right)}$

③ Find t when $P = 50{,}000$ cells

$50{,}000 = 5{,}000 e^{\frac{\ln 2.4}{10} \cdot t}$

$\frac{50{,}000}{5{,}000} = e^{\frac{\ln 2.4}{10} \cdot t}$

$\ln(10) = \frac{\ln 2.4}{10} \cdot t \cdot \cancel{\ln e}^{1}$

$\frac{10 \cdot \ln(10)}{\ln(2.4)} = t$

$t \approx 26.3$

Ans: We will have 50,000 cells in approximately 26.3 hours.

Lesson 2.10 Exponential and Logarithmic Applications

C. If certain isotope has a half-life of 4.2 days. How long will it take for a 150 milligram sample to decay so that only 10 milligrams are left?

(hint, solve for "k" first based on its given half life (t = 4.2) and then apply "k" to solve for "t" for 10 mg)

Given $P_0 = 150\text{ mg}$

① find K given half-life of 4.2 days

$P = P_0 e^{Kt}$

$75 = 150 e^{K(4.2)}$

$\frac{1}{2} = e^{4.2K}$

$\ln\left(\frac{1}{2}\right) = 4.2K \ln e$

$\frac{\ln\left(\frac{1}{2}\right)}{4.2} = K$

$K \approx -.165035$

② model: $P = 150 e^{\left(\frac{\ln(.5)}{4.2} \cdot t\right)}$

③ Find t when $P = 10\text{ mg}$

$10 = 150 e^{\left(\frac{\ln(.5)}{4.2} \cdot t\right)}$

$\frac{10}{150} = e^{\frac{\ln(.5)}{4.2} \cdot t}$

$\ln\left(\frac{1}{15}\right) = \frac{\ln(.5)}{4.2} t \cdot \cancel{\ln e}^{1}$

$\frac{4.2 \ln\left(\frac{1}{15}\right)}{\ln(.5)} = t$

$t \approx 16.4$

It will take about 16.4 days.

D. The half-life of carbon-14 is 5730 years. If it is determined that an old bone contains 85% of it original carbon-14 how old is the bone?

Given $P = .85 P_0$

① find K given half-life of $C^{14} \rightarrow 5730$ yrs

$P = P_0 e^{Kt}$

$\frac{1}{2} P_0 = P_0 e^{K \cdot 5730}$

$\frac{1}{2} = e^{K \cdot 5730}$

$\ln\left(\frac{1}{2}\right) = K \cdot 5730 \cancel{\ln e}^{1}$

$\frac{\ln(.5)}{5730} = K$

② model: $P = P_0 e^{\left(\frac{\ln(.5)}{5730} \cdot t\right)}$

③ Find t when $P = .85 P_0$

$.85 P_0 = P_0 e^{\frac{\ln(.5)}{5730} \cdot t}$

$.85 = e^{\frac{\ln(.5)}{5730} \cdot t}$

$\ln(.85) = \frac{\ln(.5)}{5730} \cdot t \ln e$

$\frac{5730 \ln(.85)}{\ln(.5)} = t$

$t \approx 1{,}343.5$

Ans: It takes about 1,343.5 yrs for a bone to lose 15% of its carbon-14.