Lesson 2.10 Exponential and Logarithmic Applications

Learning Objectives - SWBAT:

1. Solve real world problems involving Exponential and Logarithmic principles
   > Population Growth, The Richter Scale, Compound Interest

Example 1 Population Growth

Estimates of the world population (in millions) from 1998 through 2007 are shown in the table. A scatter plot of the data is shown in Figure 3.43. (Source: U.S. Bureau of the Census)

Year	Population, P	Year	Population, P
1998	5930	2003	6303
1999	6006	2004	6377
2000	6082	2005	6451
2001	6156	2006	6525
2002	6230	2007	6600

An exponential growth model that approximates this data is given by

$$P = 5400e^{0.011852t}, \qquad 8 \le t \le 17$$

where P is the population (in millions) and $t = 8$ represents 1998. Compare the values given by the model with the estimates shown in the table. According to this model, when will the world population reach 6.8 billion?

Solution To find when the world population will reach 6.8 billion, let $P = 6800$ in the model and solve for t.

$5400e^{0.011852t} = P$ — Write original model.

$5400e^{0.011852t} = 6800$ — Substitute 6800 for P.

$e^{0.011852t} \approx 1.25926$ — Divide each side by 5400.

$\ln e^{0.011852t} \approx \ln 1.25926$ — Take natural log of each side.

$0.011852t \approx 0.23052$ — Inverse Property

$t \approx 19.4$ — Divide each side by 0.011852.

According to the model, the world population will reach 6.8 billion in 2009.

Your Turn

Population The populations P (in thousands) of Pittsburgh, Pennsylvania from 1990 to 2004 can be modeled by $P = 372.55e^{-0.01052t}$, where t is the year, with $t = 0$ corresponding to 1990. (Source: U.S. Census Bureau)

(a) According to the model, was the population of Pittsburgh increasing or decreasing from 1990 to 2004? Explain your reasoning.

(b) What were the populations of Pittsburgh in 1990, 2000, and 2004?

(c) According to the model, when will the population be approximately 300,000?

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The Richter Scale **Magnitudes of Earthquakes**

On the Richter scale, the magnitude R of an earthquake of intensity I is given by

$$R = \log_{10} \frac{I}{I_0}$$

where $I_0 = 1$ is the minimum intensity used for comparison. Intensity is a measure of the wave energy of an earthquake.

Example 2 In 2001, the coast of Peru experienced an earthquake that measured 8.4 on the Richter scale. In 2003, Colima, Mexico experienced an earthquake that measured 7.6 on the Richter scale. Find the intensity of each earthquake and compare the two intensities.

Solution

Because $I_0 = 1$ and $R = 8.4$, you have

$$8.4 = \log_{10} \frac{I}{1} \qquad \text{Substitute 1 for } I_0 \text{ and 8.4 for } R.$$

$$10^{8.4} = 10^{\log_{10} I} \qquad \text{Exponentiate each side.}$$

$$10^{8.4} = I \qquad \text{Inverse Property}$$

$$251{,}189{,}000 \approx I. \qquad \text{Use a calculator.}$$

For $R = 7.6$, you have

$$7.6 = \log_{10} \frac{I}{1} \qquad \text{Substitute 1 for } I_0 \text{ and 7.6 for } R.$$

$$10^{7.6} = 10^{\log_{10} I} \qquad \text{Exponentiate each side.}$$

$$10^{7.6} = I \qquad \text{Inverse Property}$$

$$39{,}811{,}000 \approx I. \qquad \text{Use a calculator.}$$

Note that an increase of 0.8 unit on the Richter scale (from 7.6 to 8.4) represents an increase in intensity by a factor of

$$\frac{251{,}189{,}000}{39{,}811{,}000} \approx 6.$$

In other words, the 2001 earthquake had an intensity about 6 times as great as that of the 2003 earthquake.

Your Turn

41. Find the intensities I of the following earthquakes measuring R on the Richter scale (let $I_0 = 1$). (Source: U.S. Geological Survey)

(a) Santa Cruz Islands in 2006, $R = 6.1$

(b) Pakistan in 2005, $R = 7.6$

(c) Northern Sumatra in 2004, $R = 9.0$

Lesson 2.10 Exponential and Logarithmic Applications

Compound Interest - In lesson 2.2 we learned how to use the compound interest formula and the continuously compounding interest formula to determine the future value of an investment. Now that we know about the properties of exponential and logarithmic equations, we can now solve for all of the other variables in these formulas:

Compound Interest

$$FV = P\left(1+\frac{r}{n}\right)^{nt}$$

FV = future value of the deposit
P = principal or amount of money deposited
r = annual interest rate (in decimal form)
n = number of times compounded per year
t = time in years.

Continuously compounding Interest

$$FV = Pe^{rt}$$

Example 3 - If you deposit \$5,000 into an account that pays 6% interest compounded monthly, how long would it take for the account grow to \$8,000

$$8000 = 5000\left(1+\frac{0.06}{12}\right)^{12t}$$

Plug in the giving information, FV = 8000, P = 5000, r = 0.06, and n = 12.

$$8000 = 5000(1.005)^{12t}$$

Use the order or operations to simplify the problem. Keep as many decimals as possible until the final step.

$$1.6 = 1.005^{12t}$$

Divide each side by 5000.

$$\log(1.6) = \log(1.005^{12t})$$

$$\log 1.6 = (12t)(\log 1.005)$$

Take the logarithm of each side. Then use Property 5 to rewrite the problem as multiplication.

$$\frac{\log 1.6}{\log 1.005} = 12t$$

Divide each side by log 1.005.

$$94.23553232 \approx 12t$$

Use a calculator to find log 1.6 divided by log 1.005.

$$t \approx 7.9$$

Finish solving the problem by dividing each side by 12 and round your final answer.

It will take approximately 7.9 years for the account to go from \$5000 to \$8000.

Your Turn - If you deposit \$8,000 into an account that pays 7% interest compounded quarterly, how long would it take for the account grow to \$12,400

Lesson 2.10 Exponential and Logarithmic Applications

Practice ***Population*** The population P (in thousands) of Reno, Nevada can be modeled by

$$P = 134.0e^{kt}$$

where t is the year, with $t = 0$ corresponding to 1990. In 2000, the population was 180,000. (Source: U.S. Census Bureau)

(a) Find the value of k for the model. Round your result to four decimal places.

(b) Use your model to predict the population in 2010.

Population The population P (in thousands) of Las Vegas, Nevada can be modeled by

$$P = 258.0e^{kt}$$

where t is the year, with $t = 0$ corresponding to 1990. In 2000, the population was 478,000. (Source: U.S. Census Bureau)

(a) Find the value of k for the model. Round your result to four decimal places.

(b) Use your model to predict the population in 2010.

42. Find the magnitudes R of the following earthquakes of intensity I (let $I_0 = 1$).

(a) $I = 39{,}811{,}000$

(b) $I = 12{,}589{,}000$

(c) $I = 251{,}200$

Lesson 2.10 Exponential and Logarithmic Applications

Practice

Compound Interest **In Exercises 7–14, complete the table for a savings account in which interest is compounded continuously.**

	Initial Investment	*Annual % Rate*	*Time to Double*	*Amount After 10 Years*
7.	$10,000	3.5%		
8.	$2000	1.5%		
9.	$7500		21 years	
10.	$1000		12 years	
11.	$5000			$5665.74
12.	$300			$385.21
13.		4.5%		$100,000.00
14.		2%		$2500.00

Problem 1: If you deposit $4500 at 5% annual interest compounded quarterly, how much money will be in the account after 10 years?

Problem 2: If you deposit $4000 into an account paying 9% annual interest compounded monthly, how long until there is $10000 in the account?

Problem 3: If you deposit $2500 into an account paying 11% annual interest compounded quarterly, how long until there is $4500 in the account?

Problem 4: How much money would you need to deposit today at 5% annual interest compounded monthly to have $20000 in the account after 9 years?

Problem 5: If you deposit $6000 into an account paying 6.5% annual interest compounded quarterly, how long until there is $12600 in the account?

Problem 6: If you deposit $5000 into an account paying 8.25% annual interest compounded semiannually, how long until there is $9350 in the account?

Lesson 2.10 Exponential and Logarithmic Applications

Exponential Growth / Decay Model – Given exponential growth or decay the amount P after time t is given by the following formula: $P = P_0 e^{kt}$

Here P_0 is the initial amount and k is the exponential growth/decay rate.

If k is positive then we will have a growth model and if k is negative then we will have a decay model.

Use the exponential growth/decay model to answer the questions.

A. A certain bacterium has an exponential growth rate of 25% per day. If we start with 0.5 gram and provide unlimited resources how much bacteria can we grow in 2 weeks?

B. During its exponential growth phase, a certain bacterium can grow from 5,000 cells to 12,000 cells in 10 hours. At this rate how long will it take to grow to 50,000 cells?

C. If certain isotope has a half-life of 4.2 days. How long will it take for a 150 milligram sample to decay so that only 10 milligrams are left?

(hint, solve for "k" first based on its given half life (t = 4.2) and then apply "k" to solve for "t" for 10 mg)

D. The half-life of carbon-14 is 5730 years. If it is determined that an old bone contains 85% of it original carbon-14 how old is the bone?