Lesson 2.1 - Exponential Functions and their graphs

Learning Objectives - SWBAT:

1. Define and Evaluate Exponential Functions
2. Identify key attributes of exponential function graphs
3. Identify transformations from the parent exponential function given an equation
4. Write the equation of an exponential function given its transformations

Overview:

So far, this text has dealt mainly with **algebraic functions,** which include polynomial functions and rational functions. In this chapter you will study two types of nonalgebraic functions—*exponential functions* and *logarithmic functions*. These functions are examples of **transcendental functions.**

> **Definition of Exponential Function**
>
> The **exponential function *f* with base *a*** is denoted by
>
> $$f(x) = a^x$$
>
> where $a > 0$, $a \neq 1$, and x is any real number.

Note that in the definition of an exponential function, the base $a = 1$ is excluded because it yields $f(x) = 1^x = 1$. This is a constant function, not an exponential function.

You have already evaluated a^x for integer and rational values of x. For example, you know that $4^3 = 64$ and $4^{1/2} = 2$. However, to evaluate 4^x for any real number x, you need to interpret forms with *irrational* exponents. For the purposes of this text, it is sufficient to think of

$$a^{\sqrt{2}} \left(\text{where } \sqrt{2} \approx 1.41421356\right)$$

as the number that has the successively closer approximations

$$a^{1.4}, a^{1.41}, a^{1.414}, a^{1.4142}, a^{1.41421}, \ldots .$$

Example 1 shows how to use a calculator to evaluate exponential functions.

Example 1: Evaluating Exponential Functions

Use a calculator to evaluate each function at the indicated value of x.

	Function	*Value*
a.	$f(x) = 2^x$	$x = -3.1$
b.	$f(x) = 2^{-x}$	$x = \pi$
c.	$f(x) = 0.6^x$	$x = \frac{3}{2}$

Solution

	Function Value	*Graphing Calculator Keystrokes*	*Display*
a.	$f(-3.1) = 2^{-3.1}$	2 [^] [(−)] 3.1 [ENTER]	0.1166291
b.	$f(\pi) = 2^{-\pi}$	2 [^] [(−)] π [ENTER]	0.1133147
c.	$f(\frac{3}{2}) = (0.6)^{3/2}$	.6 [^] [(] 3 [÷] 2 [)] [ENTER]	0.4647580

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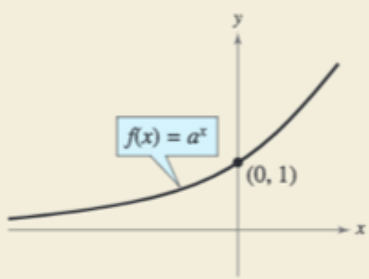
Library of Parent Functions: Exponential Function

The *exponential function*

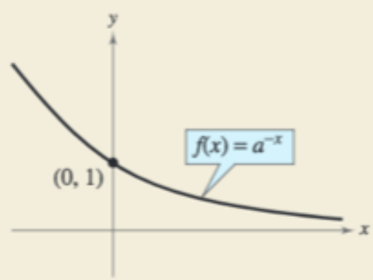
$$f(x) = a^x, \ a > 0, \ a \neq 1$$

is different from all the functions you have studied so far because the variable x is an *exponent*. A distinguishing characteristic of an exponential function is its rapid increase as x increases (for $a > 1$). Many real-life phenomena with patterns of rapid growth (or decline) can be modeled by exponential functions. The basic characteristics of the exponential function are summarized below. A review of exponential functions can be found in the *Study Capsules*.

Graph of $f(x) = a^x, \ a > 1$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
x-axis is a horizontal asymptote
$(a^x \to 0$ as $x \to -\infty)$
Continuous



Graph of $f(x) = a^{-x}, \ a > 1$
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Decreasing on $(-\infty, \infty)$
x-axis is a horizontal asymptote
$(a^{-x} \to 0$ as $x \to \infty)$
Continuous



Transformations of $g(x) = b^x$ $(c > 0)$: **(Order of transformations is H S R V.)**

Horizontal:
$g(x) = b^{x+c}$ (graph moves c units left)
$g(x) = b^{x-c}$ (graph moves c units right)

Stretch/Shrink: (Vertical)
$g(x) = cb^x$ (graph stretches if $c > 1$)
(graph shrinks if $0 < c < 1$)

Stretch/Shrink: (Horizontal)
$g(x) = b^{cx}$ (graph shrinks if $c > 1$)
(graph stretches if $0 < c < 1$)

Reflection:
$g(x) = -b^x$ (graph reflects over the x-axis)
$g(x) = b^{-x}$ (graph reflects over the y-axis)

Vertical:
$g(x) = b^x + c$ (graph moves up c units)
$g(x) = b^x - c$ (graph moves down c units)

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Transformations of Exponential Functions

Each of the following graphs is a transformation of the graph of $f(x) = 3^x$.

a. Because $g(x) = 3^{x+1} = f(x + 1)$, the graph of g can be obtained by shifting the graph of f one unit to the *left*, as shown in Figure 3.5.

b. Because $h(x) = 3^x - 2 = f(x) - 2$, the graph of h can be obtained by shifting the graph of f *downward* two units, as shown in Figure 3.6.

c. Because $k(x) = -3^x = -f(x)$, the graph of k can be obtained by *reflecting* the graph of f in the x-axis, as shown in Figure 3.7.

d. Because $j(x) = 3^{-x} = f(-x)$, the graph of j can be obtained by *reflecting* the graph of f in the y-axis, as shown in Figure 3.8.

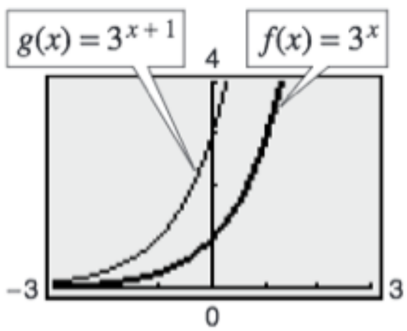

Figure 3.5

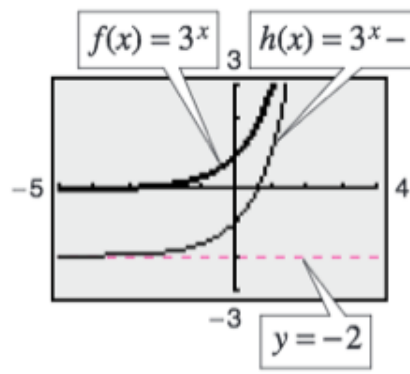

Figure 3.6

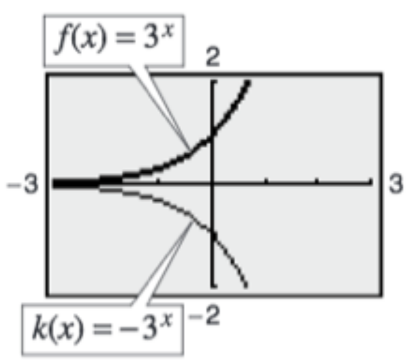

Figure 3.7

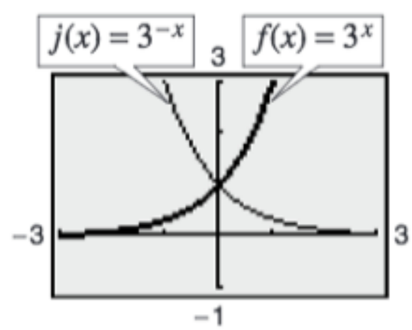

Figure 3.8

Notice that the transformations in Figures 3.5, 3.7, and 3.8 keep the x-axis ($y = 0$) as a horizontal asymptote, but the transformation in Figure 3.6 yields a new horizontal asymptote of $y = -2$. Also, be sure to note how the y-intercept is affected by each transformation.

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Practice

State the domain and range of the following exponential functions:

(a) $f(x) = 2^x$ domain: ___________ range: ___________

(b) $g(x) = \left(\frac{1}{2}\right)^x$ domain: ___________ range: ___________

3. $f(x) = 3^x$ domain: ___________ range: ___________

4. $f(x) = -(3^x)$ domain: ___________ range: ___________

5. $f(x) = 3^{-x}$ domain: ___________ range: ___________

6. $f(x) = \left(\frac{1}{3}\right)^x$ domain: ___________ range: ___________

Identify each transformation from the parent function of f(x)=B^x. Tell if the function is a decay or growth function.

1. g(x)= 3^{x-2} ______________________________
2. g(x) = $\frac{1}{2}^x + 3$ ______________________________
3. g(x) = $-4^x - 6$ ______________________________
4. g(x)= $-\frac{2^{x-5}}{3} + 4$ ______________________________
5. g(x)= $2^{x-7} + 5$ ______________________________
6. g(x)= $3(2^{x+1}) + 2$ ______________________________

Write the function for each graph described below.

7. the graph of f(x) = 2^x, reflected across the x axis. ______________
8. The graph of f(x)=$\frac{1}{3}^x$, translated up 5 units. ______________
9. The graph of f(x) = 3^x, left 2 units, and down 3. ______________
10. The graph of f(x) = $\frac{1}{2}^x$, translated down 2 units ______________
11. The graph of f(x) = 4^x, stretched horizontally by a factor of 3 ______________
12. The graph of f(x) = 2^x, up 4 units, right 3 ______________

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Practice

1) Describe the transformations that map the function $y = 2^x$ onto each of the following functions...

a) $y = 2^x - 2$ **b)** $y = 2^{x+3}$ **c)** $y = 4^x$ **d)** $y = 3(2^{x-1}) + 1$

3) Write the equation for the function that results from each transformation applied to the base function $y = 5^x$.

a) translate down 3 units

b) shift right 2 units

c) translate left $\frac{1}{2}$ unit

d) shift up 1 unit and left 2.5 units

4) Write the equation for the function that results from each transformation applied to the base function $f(x) = \left(\frac{1}{3}\right)^x$

a) reflect in the x- axis (vertical reflection)

b) stretch vertically by a factor of 3

c) stretch horizontally by a factor of 2.4

d) reflect horizontally, stretch vertically by factor of 4

Without a calculator, match each function with its graph.

_____1] $f(x) = 5^x$ _____2] $f(x) = -5^{-x}$ _____3] $f(x) = -5^{x-1}$

_____4] $f(x) = 5^{-x} + 1$ _____5] $f(x) = 5^{x+1}$ _____6] $f(x) = -5^x - 1$