Lesson 2.4 - Graphs and Properties of Logarithms

Learning Objective: SWBAT:

1. Describe the relationship between an Exponential function and a Logarithmic Function (graphically)
2. Describe transformations of logarithmic Graphs
3. Use the basic properties of logs to simplify expressions

Notes

Library of Parent Functions: Logarithmic Function

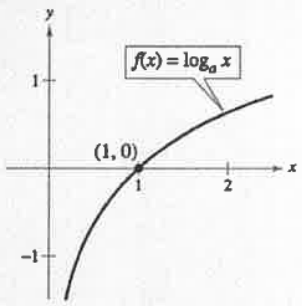
The *logarithmic function*

$$f(x) = \log_a x, \quad a > 0, \ a \neq 1$$

is the inverse function of the exponential function. Its domain is the set of positive real numbers and its range is the set of all real numbers. This is the opposite of the exponential function. Moreover, the logarithmic function has the y-axis as a vertical asymptote, whereas the exponential function has the x-axis as a horizontal asymptote. Many real-life phenomena with a slow rate of growth can be modeled by logarithmic functions. The basic characteristics of the logarithmic function are summarized below. A review of logarithmic functions can be found in the *Study Capsules.*

Graph of $f(x) = \log_a x, \ a > 1$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
y-axis is a vertical asymptote
($\log_a x \to -\infty$ as $x \to 0^+$)
Continuous
Reflection of graph of $f(x) = a^x$ in the line $y = x$



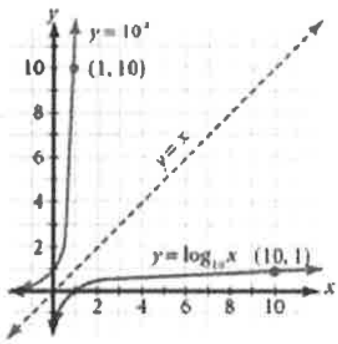
More about the inverse relationship between exponential and logarithmic functions:

The inverse of the exponential function $y = 10^x$ is $x = 10^y$. To rewrite $x = 10^y$ in terms of y, use the equivalent logarithmic form, $y = \log_{10} x$.

Examine the tables and graphs below to see the inverse relationship between $y = 10^x$ and $y = \log_{10} x$.

x	$y = 10^x$
-3	$\frac{1}{1000}$
-2	$\frac{1}{100}$
-1	$\frac{1}{10}$
0	1
1	10
2	100
3	1000

x	$y = \log_{10} x$
$\frac{1}{1000}$	-3
$\frac{1}{100}$	-2
$\frac{1}{10}$	-1
1	0
10	1
100	2
1000	3



The table below summarizes the relationship between the domain and range of $y = 10^x$ and of $y = \log_{10} x$.

Function	Domain	Range
$y = 10^x$	all real numbers	all positive real numbers
$y = \log_{10} x$	all positive real numbers	all real numbers

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Transformations of Logarithmic Function Graphs

Transformation	$f(x)$ Notation	Examples	
Vertical translation	$f(x)+k$	$y = \log x + 3$ $y = \log x - 4$	3 units up 4 units down
Horizontal translation	$f(x-h)$	$y = \log(x-2)$ $y = \log(x+1)$	2 units right 1 unit left
Vertical stretch or compression	$af(x)$	$y = 6\log x$ $y = \frac{1}{2}\log x$	stretch by 6 compression by $\frac{1}{2}$
Horizontal stretch or compression	$f\left(\frac{1}{b}x\right)$	$y = \log\left(\frac{1}{5}x\right)$ $y = \log(3x)$	stretch by 5 compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\log x$ $y = \log(-x)$	across x-axis across y-axis

Basic Properties of Logarithms (with examples)

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$.
2. $\log_a a = 1$ because $a^1 = a$.
3. $\log_a a^x = x$ and $a^{\log_a x} = x$. — Inverse Properties
4. If $\log_a x = \log_a y$, then $x = y$. — One-to-One Property

Example 3 Using Properties of Logarithms

a. Solve for x: $\log_2 x = \log_2 3$

b. Solve for x: $\log_4 4 = x$

c. Simplify: $\log_5 5^x$

d. Simplify: $7^{\log_7 14}$

Solution

a. Using the One-to-One Property (Property 4), you can conclude that $x = 3$.

b. Using Property 2, you can conclude that $x = 1$.

c. Using the Inverse Property (Property 3), it follows that $\log_5 5^x = x$.

d. Using the Inverse Property (Property 3), it follows that $7^{\log_7 14} = 14$.

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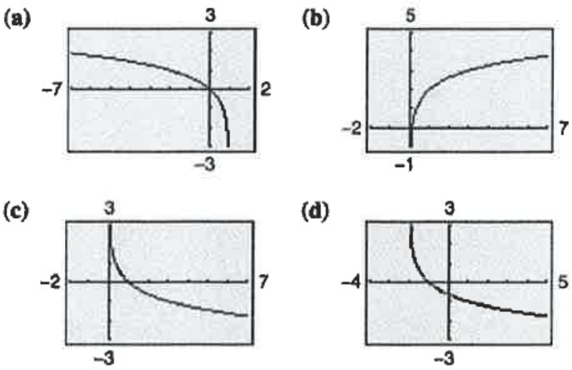
Practice

Library of Parent Functions **In Exercises 53–56, use the graph of $y = \log_3 x$ to match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]**



B **53.** $f(x) = \log_3 x + 2$ **54.** $f(x) = -\log_3 x$ C

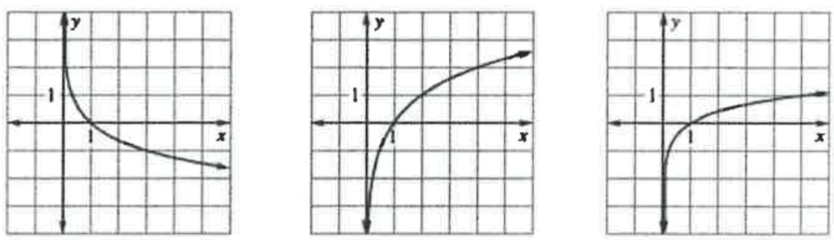
D **55.** $f(x) = -\log_3(x + 2)$ **56.** $f(x) = \log_3(1 - x)$ A

Match the function with its graph.

1. $f(x) = \log_2 x$ **2.** $f(x) = \log_5 x$ **3.** $f(x) = \log_{1/3} x$

B C A

A. **B.** **C.**



For problems 4 - 11: Describe the transformations present from the parent function AND state the domain/range of each function

4. $f(x) = \log_3 x$

D: $(0,\infty)$, R: $(-\infty,\infty)$

no transformations

5. $f(x) = \log_3(x + 2)$

D: $(-2,\infty)$, R: $(-\infty,\infty)$

Translation left 2

6. $f(x) = -\log_3 x - 1$

D: $(0,\infty)$, R: $(-\infty,\infty)$

Translation down 1

Reflect over x

7. $f(x) = \log_2(x-3)+1$

D: $(3,\infty)$, R: $(-\infty,\infty)$

Translation right 3, up 1

8. $-\log_3(x+1)$

D: $(-1,\infty)$, R: $(-\infty,\infty)$

Reflection over x

translation left 1

9. $f(x) = 3\log_2 x - 4$

D: $(0,\infty)$, R: $(-\infty,\infty)$

V. stretch by factor of 3

Translation down 4

10. $f(x) = 4\log_{1/3}(x+2)$

D: $(-2,\infty)$, R: (∞,∞)

v stretch by factor of 4

translation left 2

11. $-\log_{1/2} x + 3$

D: $(0,\infty)$, R: $(-\infty,\infty)$

Reflection over x

translation up 3

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Practice

In Exercises 33–38, solve the equation for x.

$x=9$ **33.** $\log_7 x = \log_7 9$ **34.** $\log_5 5 = x$ $x=1$

$x=2$ **35.** $\log_6 6^2 = x$ **36.** $\log_2 2^{-1} = x$ $x=-1$

$x=\frac{1}{10}$ **37.** $\log_8 x = \log_8 10^{-1}$ **38.** $\log_4 4^3 = x$ $x=3$

In Exercises 39–42, use the properties of logarithms to rewrite the expression.

$3x$ **39.** $\log_4 4^{3x}$ **40.** $6^{\log_6 36}$ 36

-3 **41.** $3 \log_2 \frac{1}{2}$ **42.** $\frac{1}{4} \log_4 16$ $\frac{1}{2}$

In Exercises 57–62, use the graph of f to describe the transformation that yields the graph of g.

57. $f(x) = \log_{10} x, \quad g(x) = -\log_{10} x$ - reflection over x

58. $f(x) = \log_{10} x, \quad g(x) = \log_{10}(x + 7)$ - translate left 7

59. $f(x) = \log_2 x, \quad g(x) = 4 - \log_2 x$ - reflect over x and translate up 4

60. $f(x) = \log_2 x, \quad g(x) = 3 + \log_2 x$ - translate 3 up

61. $f(x) = \log_8 x, \quad g(x) = -2 + \log_8(x + 3)$ - translate left 3, down 2

62. $f(x) = \log_8 x, \quad g(x) = 4 + \log_8(x - 1)$ - translate right 1, up 4

Simplify each expression.

31) $12^{\log_{12} 144}$ $= 144$ 32) $5^{\log_5 17}$ $= 17$

33) $x^{\log_x 72}$ $= 72$ 34) $9^{\log_3 20}$ $= 400$