Lesson 2.6 - The change of Base Formula

Learning Objectives - SWBAT:

1. Use the change of base formula to evaluate logarithms that are not in base 10

Making a connection

- Thus far, we have learned how to evaluate logarithms that are either base 10 OR that are "nice" to work with
  - > For example: $\log_2 8 = 3$ because $2^3 = 8$ (the numbers workout "nicely")
- As we all know, not all numbers play nicely.
  - > For example, **what is $\log_2 15$?**
    - – We know that we can raise 2 to some (unknown) power to get 15
    - – We also know that this power must be between 3 and 4 because $2^3 = 8$ and $2^4 = 16$ (and 15 is between 8 and 16)
- For these situations, we use the **change of base formula** to determine the exponent of a log that is not an integer

Change-of-Base Formula

For any positive real numbers $a \neq 1$, $b \neq 1$, and $x > 0$:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

- Example: **what is $\log_2 15$**
  - > When using the formula, it does not matter what base you use as long as you use the same base for the top and bottom
  - > For our purposes, I recommend you use base 10, which is just the default log on your calculator
    - « Set up using formula: $\frac{\log 15}{\log 2}$ Then put in calculator... = 3.90689059561
  - > What the answer "means" is that $2^{3.90689059561} = 15$ (please check using calc.)
  - > For our purposes, **please round three decimal places**

Practice

Use a calculator to approximate each to the nearest thousandth.

1) $\log_3 3.3$ $\frac{\log 3.3}{\log 3} = 1.087$

2) $\log_2 30$ $\frac{\log 30}{\log 2} = 4.907$

3) $\log_4 5$ $\frac{\log 5}{\log 4} = 1.1$

4) $\log_2 2.1$ $\frac{\log 2.1}{\log 2} = 1.07$

5) $\log 3.55$ .055

6) $\log_6 13$ 1.432

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Practice

7) $\log_6 40$

2.059

8) $\log_4 3.5$

.904

9) $\log_2 2.9$

1.536

10) $\log_6 22$

1.725

11) $\log_7 8.7$

1.112

12) $\log_3 62$

3.757

15) $\log_2 8.7$

3.121

16) $\log_9 71$

1.94

19) $\log_{13} 12.9$

.997

20) $\log_5 10.818$

1.48

21) $\log_3 189$

4.771

22) $\log_{16} 194$

1.9

23) $\log_5 183$

3.237

24) $\log_{14} 2.6$

.362