Lesson 2.7 - Properties of Logarithmic Operations

Learning Objectives - SWBAT:

1. Use the properties of Logarithmic operations to expand/condense logarithmic expressions

Making a connection

- Logarithms have rules as well that allows us to operate with them. These laws are very similar to the laws of exponents (with makes sense since logarithms are the inverse of exponential expressions
- The laws of logarithms are as follows:

1. $\boxed{\log_a AB = \log_a A + \log_a B.}$ (The logarithm of a product is the sum of the logarithms.)

2. $\boxed{\log_a \left(\frac{A}{B}\right) = \log_a A - \log_a B.}$ (The logarithm of a quotient is the difference of the logarithms.)

3. $\boxed{\log_a A^n = n \log_a A.}$ (The logarithm of a quantity raised to a power is the same as the power times the logarithm of the quantity.)

- Using these laws, we can expand/condense (re-arrange) logarithmic expressions

Example #1 - The Product Rule

- **Expand the following:** $\log_3 (2x)$
  - > This is a multiplication problem since 2 is multiplying x so the product rule applies
  - > The expanded form using the product rule would be $\boxed{\log_3 x + \log_3 2}$
  - > If the problem is given in expanded form, follow the same pattern to "condense" the logs

Example #2 - The Quotient Rule

- **Expand the following:** $\log_4\left(\frac{4}{x}\right)$
  - > This is a division problem so the quotient rule applies
  - > The expanded form using the quotient rule would be $\boxed{\log_4 4 - \log_4 x}$

Example #3 - The Power Rule

- **Expand the following:** $\log_5 (x^3)$
  - > Since the x is being raised to a power, the power rule applies
  - > The expanded form using the quotient rule would be $\boxed{3 \log_5 x}$

Example #4 - Multiple laws

- **Expand the following** $\log_2\left(\frac{8x^4}{5}\right)$
  - > Since the parenthesis says the entire term is being divided, use the quotient rule first in expanding the logarithm. The result is: $\log_2(8x^4) - \log_2(5)$
  - > The term on the right is as expanded as it can get, but the term on the left still has both multiplication and an exponent. Use the product rule to expand the term on the left: $\log_2(8) + \log_2(x^4) - \log_2(5)$
  - > Lastly, the middle term still has an exponent. Use the power rule to expand this term and get the final answer: $\boxed{3 + 4\log_2(x) - \log_2(5)}$

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Your Turn #1 - Expand each Logarithm

$\log 10x$

$\log_{10} 10 + \log_{10} x$

$\boxed{1 + \log x}$

$\log_b \frac{x^4}{z^2}$

$\log_b x^4 - \log_b z^2$

$\boxed{4\log_b - 2\log_b z}$

$\log_4 4x^2$

$\log_4 4 + \log_4 x^2$

$\boxed{1 + 2\log_4 x}$

$\log_3 \sqrt{x-2} = \log_3 (x-2)^{1/2}$

$\boxed{\frac{1}{2}\log_3 (x-2)}$

Your Turn #2 - Condense each Logarithm

$\log 7 - \log x$

$\boxed{\log\left(\frac{7}{x}\right)}$

$\log_2 5 + \log_2 x - \log_2 3$

$\log_2 5x - \log_2 3$

$\boxed{\log_2\left(\frac{5x}{3}\right)}$

$\frac{1}{2}\log_5 7 - 2\log_5 x$

$\log_5 7^{1/2} - \log_5 x^2$

$\log_5 \frac{7^{1/2}}{x^2} = \boxed{\log_5 \frac{\sqrt{7}}{x^2}}$

$1 + 3\log_4 x$

$\log_4 4 + 3\log_4 x$

$\log_4 4 + \log_4 x^3$

$\boxed{\log_4 (4x^3)}$

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Practice

Expand each logarithm.

1) $\log(6 \cdot 11)$

$\log 6 + \log 11$

2) $\log(5 \cdot 3)$

$\log 5 + \log 3$

3) $\log\left(\frac{6}{11}\right)^5$

$5\log 6 - 5\log 11$

4) $\log(3 \cdot 2^3)$

$\log 3 + 3\log 2$

5) $\log \frac{2^4}{5}$

$4\log 2 - \log 5$

6) $\log\left(\frac{6}{5}\right)^6$

$\log 6^6 - \log 5^6$

$6\log 6 - 6\log 5$

7) $\log \frac{x}{y^6}$

$\log x - 6\log y$

8) $\log(a \cdot b)^2$

$2\log a + 2\log b$

9) $\log \frac{u^4}{v}$

$4\log u - \log v$

10) $\log \frac{x}{y^5}$

$\log x - 5\log y$

11) $\log \sqrt[3]{x \cdot y \cdot z} = \log(xyz)^{1/3}$

$\frac{\log x + \log y + \log z}{3}$

12) $\log(x \cdot y \cdot z^2)$

$\log x + \log y + 2\log z$

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Practice

Condense each expression to a single logarithm.

13) $\log 3 - \log 8$

$\log\left(\frac{3}{8}\right)$

14) $\frac{\log 6}{3}$

$= \frac{1}{3}\log 6$

$= \log 6^{1/3}$

$= \log \sqrt[3]{6}$

15) $4\log 3 - 4\log 8$

$\log\left(\frac{3}{8}\right)^4$

16) $\log 2 + \log 11 + \log 7$

$\log(2 \cdot 11 \cdot 7) = \log 154$

17) $\log 7 - 2\log 12$

$\log\left(\frac{7}{12^2}\right) = \log\frac{7}{144}$

18) $\frac{2\log 7}{3}$

$\frac{1}{3}\log 7^2 = \frac{1}{3}\log 49$

$= \log\sqrt[3]{49}$

19) $6\log_3 u + 6\log_3 v$

$\log_3(u^6 v^6)$

20) $\ln x - 4\ln y$

$\ln\frac{x}{y^4}$

21) $\log_4 u - 6\log_4 v$

$\log_4\frac{u}{v^6}$

22) $\log_3 u - 5\log_3 v$

$\log_3\left(\frac{u}{v^5}\right)$

23) $20\log_6 u + 5\log_6 v$

$\log_6(v^5 u^{20})$

24) $4\log_3 u - 20\log_3 v$

$\log_3\left(\frac{u^4}{v^{6}}\right)$

25) $2(\log 2x - \log y) - (\log 3 + 2\log 5)$

$\log\left(\frac{4x^2}{75y^2}\right)$

26) $\log x \cdot \log 2$

is already simplfied as much as it can go