Lesson 2.8 - Solving Exponential Equations

Learning Objectives - SWBAT:

1. Solve Exponential equations that both do and do not require logarithms

So far in this chapter, you have studied the definitions, graphs, and properties of exponential and logarithmic functions. In this section, you will study procedures for *solving equations* involving exponential and logarithmic functions.

There are two basic strategies for solving exponential or logarithmic equations. The first is based on the One-to-One Properties and the second is based on the Inverse Properties. For $a > 0$ and $a \neq 1$, the following properties are true for all x and y for which $\log_a x$ and $\log_a y$ are defined.

One-to-One Properties

$a^x = a^y$ if and only if $x = y$.

$\log_a x = \log_a y$ if and only if $x = y$.

Inverse Properties

$a^{\log_a x} = x$

$\log_a a^x = x$

Example 1 **Solving Simple Exponential and Logarithmic Equations**

	Original Equation	*Rewritten Equation*	*Solution*	*Property*
a.	$2^x = 32$	$2^x = 2^5$	$x = 5$	One-to-One
b.	$\ln x - \ln 3 = 0$	$\ln x = \ln 3$	$x = 3$	One-to-One
c.	$\left(\frac{1}{3}\right)^x = 9$	$3^{-x} = 3^2$	$x = -2$	One-to-One
d.	$e^x = 7$	$\ln e^x = \ln 7$	$x = \ln 7$	Inverse
e.	$\ln x = -3$	$e^{\ln x} = e^{-3}$	$x = e^{-3}$	Inverse
f.	$\log_{10} x = -1$	$10^{\log_{10} x} = 10^{-1}$	$x = 10^{-1} = \frac{1}{10}$	Inverse

Your Turn solve the exponential equation.

17. $4^x = 16$

$x = 2$

18. $3^x = 243$

$x = 5$

19. $5^x = \frac{1}{625}$

$x = -4$

20. $7^x = \frac{1}{49}$

$x = -2$

27. $2^{x+3} = 256$

$x = 5$

28. $3^{x-1} = \frac{1}{81}$

$x = -3$

1. $4^{2x-7} = 64$

$x = 5$

2. $2^{3x+1} = 32$

$x = \frac{4}{3}$

Strategies for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions.
2. Rewrite an *exponential* equation in logarithmic form and apply the Inverse Property of logarithmic functions.
3. Rewrite a *logarithmic* equation in exponential form and apply the Inverse Property of exponential functions.

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<u>Practice 1</u> - Solve the exponential equation. Hint, you will not need logarithms to determine answers for this problems set.

1) $5^{3n} = 125$

$n=1$

2) $2^{2n} = 16$

$n=2$

3) $5^{3r} = 5^{-2r}$

$r=0$

4) $3^{-2k} = 81$

$k=-2$

5) $2^{-3x} = 2^{x-1}$

$x=\frac{1}{4}$

6) $6^{3m} = 36$

$m=\frac{2}{3}$

7) $10^{3x} = 10000$

$x=\frac{4}{3}$

8) $4^{r+1} = 1$

$r=-1$

9) $\left(\frac{1}{8}\right)^{3x} \cdot 64^{2x+1} = 4$

$x=\frac{-4}{3}$

10) $32^{2x} = 8$

$x=\frac{3}{10}$

11) $6^{-3v-2} = 36$

$v=\frac{-4}{3}$

12) $243^{x} = 81$

$x=\frac{4}{5}$

13) $2^{-2n} \cdot 2^{n+1} = 2^{-2n}$

$n=-1$

14) $\left(\frac{1}{16}\right)^{2a} \cdot 16^{-2a-3} = 64^{2a}$

$a=\frac{-3}{7}$

21) $27^{3x} \cdot \left(\frac{1}{9}\right)^{-x} = 243^{-x-3}$

$x=\frac{-15}{16}$

22) $\left(\frac{1}{6}\right)^{3a} \cdot 36^{-3a} = \frac{1}{36}$

$a=\frac{2}{9}$

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Example 2 Solving Exponential Equations

Solve each equation. **a.** $e^x = 72$ **b.** $3(2^x) = 42$

Algebraic Solution

a.

$e^x = 72$	Write original equation.
$\ln e^x = \ln 72$	Take natural log of each side.
$x = \ln 72 \approx 4.28$	Inverse Property

The solution is $x = \ln 72 \approx 4.28$. Check this in the original equation.

b.

$3(2^x) = 42$	Write original equation.
$2^x = 14$	Divide each side by 3.
$\log_2 2^x = \log_2 14$	Take log (base 2) of each side.
$x = \log_2 14$	Inverse Property
$x = \dfrac{\ln 14}{\ln 2} \approx 3.81$	Change-of-base formula

The solution is $x = \log_2 14 \approx 3.81$. Check this in the original equation.

Your Turn

55. $2e^{5x} = 18$

$x = 0.439$

45. $8^{3x} = 360$

$x = .944$

Example 3 Solving an Exponential Equation

Solve $4e^{2x} - 3 = 2$.

Algebraic Solution

$4e^{2x} - 3 = 2$	Write original equation.
$4e^{2x} = 5$	Add 3 to each side.
$e^{2x} = \frac{5}{4}$	Divide each side by 4.
$\ln e^{2x} = \ln \frac{5}{4}$	Take natural log of each side.
$2x = \ln \frac{5}{4}$	Inverse Property
$x = \frac{1}{2} \ln \frac{5}{4} \approx 0.11$	Divide each side by 2.

The solution is $x = \frac{1}{2} \ln \frac{5}{4} \approx 0.11$. Check this in the original equation.

Your Turn

59. $7 - 2e^x = 5$

$x = 0$

60. $-14 + 3e^x = 11$

$x = 2.120$

Example 4 Solving an Exponential Equation

Solve $2(3^{2t-5}) - 4 = 11$.

Solution

$2(3^{2t-5}) - 4 = 11$	Write original equation.
$2(3^{2t-5}) = 15$	Add 4 to each side.
$3^{2t-5} = \frac{15}{2}$	Divide each side by 2.
$\log_3 3^{2t-5} = \log_3 \frac{15}{2}$	Take log (base 3) of each side.
$2t - 5 = \log_3 \frac{15}{2}$	Inverse Property
$2t = 5 + \log_3 7.5$	Add 5 to each side.
$t = \frac{5}{2} + \frac{1}{2}\log_3 7.5$	Divide each side by 2.
$t \approx 3.42$	Use a calculator.

Your Turn

49. $5(2^{3-x}) - 13 = 100$

$x = -1.498$

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Practice 2 - Solve the exponential equation. Hint, you will need logarithms to determine answers for this problems set (round 4 decimal places please).

7) $15^x = 47$

$x = 1.4217$

8) $13^r = 79$

$r = 1.7035$

9) $8 \cdot 13^{7x} = 73$

$x = .1231$

10) $-8 \cdot 9^{-5x} = -49$

$x = {}^-.165$

11) $9 \cdot 9^{k+2} = 59$

$k = -1.1442$

12) $17^{k-10} + 4 = 66$

$k = 11.4567$

13) $5^{4x} + 5 = 91$

$x = .6919$

14) $10^{-9n} + 8 = 34$

$n = {}^-.1572$

15) $e^{r+4} + 6 = 29$

$r = .8645$

16) $-10 \cdot 8^{-5n} = -81$

$n = -.2012$

17) $5.9e^{10x-8} + 0.7 = 43$

$x = .997$

18) $-3e^{5a+1} - 7 = -91$

$a = .4664$

21) $7e^{2-3x} + 3 = 92$

$x = {}^-.1809$

22) $-5e^{-6m-8} - 6 = -37$

$m = -1.6374$