Lesson 2.9 - Solving Logarithmic Equations

Learning Objectives - SWBAT:

1. Solve Logarithmic Equations

Solving Logarithmic Equations

To solve a logarithmic equation, you can write it in exponential form.

$\ln x = 3$	Logarithmic form
$e^{\ln x} = e^3$	Exponentiate each side.
$x = e^3$	Exponential form

This procedure is called ***exponentiating*** each side of an equation. It is applied after the logarithmic expression has been isolated.

Example 6 Solving Logarithmic Equations

Solve each logarithmic equation.

a. $\ln 3x = 2$ b. $\log_3(5x - 1) = \log_3(x + 7)$

Solution

a. $\ln 3x = 2$ — Write original equation.

$e^{\ln 3x} = e^2$ — Exponentiate each side.

$3x = e^2$ — Inverse Property

$x = \frac{1}{3}e^2 \approx 2.46$ — Multiply each side by $\frac{1}{3}$.

The solution is $x = \frac{1}{3}e^2 \approx 2.46$. Check this in the original equation.

b. $\log_3(5x - 1) = \log_3(x + 7)$ — Write original equation.

$5x - 1 = x + 7$ — One-to-One Property

$x = 2$ — Solve for x.

The solution is $x = 2$. Check this in the original equation.

Your Turn

87. $\ln 4x = 2.1$ $x = 2.042$

92. $\log_9(4 + x) = \log_9(2x - 1)$ $x = 5$

Example 7 Solving a Logarithmic Equation

Solve $5 + 2 \ln x = 4$.

Algebraic Solution

$5 + 2 \ln x = 4$ — Write original equation.

$2 \ln x = -1$ — Subtract 5 from each side.

$\ln x = -\frac{1}{2}$ — Divide each side by 2.

$e^{\ln x} = e^{-1/2}$ — Exponentiate each side.

$x = e^{-1/2}$ — Inverse Property

$x \approx 0.61$ — Use a calculator.

The solution is $x = e^{-1/2} \approx 0.61$. Check this in the original equation.

Your Turn

89. $-2 + 2 \ln 3x = 17$

$x = 4,453.242$

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Example 8 Solving a Logarithmic Equation

Solve $2 \log_5 3x = 4$.

Solution

$2 \log_5 3x = 4$	Write original equation.
$\log_5 3x = 2$	Divide each side by 2.
$5^{\log_5 3x} = 5^2$	Exponentiate each side (base 5).
$3x = 25$	Inverse Property
$x = \frac{25}{3}$	Divide each side by 3.

Your Turn

95. $7 \log_4(0.6x) = 12$

$x = 17.945$

96. $4 \log_{10}(x - 6) = 11$

$x = 568.341$

Example 9 Checking for Extraneous Solutions

Solve $\ln(x - 2) + \ln(2x - 3) = 2 \ln x$.

Algebraic Solution

$\ln(x - 2) + \ln(2x - 3) = 2 \ln x$	Write original equation.
$\ln[(x - 2)(2x - 3)] = \ln x^2$	Use properties of logarithms.
$\ln(2x^2 - 7x + 6) = \ln x^2$	Multiply binomials.
$2x^2 - 7x + 6 = x^2$	One-to-One Property
$x^2 - 7x + 6 = 0$	Write in general form.
$(x - 6)(x - 1) = 0$	Factor.
$x - 6 = 0 \Rightarrow x = 6$	Set 1st factor equal to 0.
$x - 1 = 0 \Rightarrow x = 1$	Set 2nd factor equal to 0.

Finally, by checking these two "solutions" in the original equation, you can conclude that $x = 1$ is not valid. This is because when $x = 1$, $\ln(x - 2) + \ln(2x - 3) = \ln(-1) + \ln(-1)$, which is invalid because -1 is not in the domain of the natural logarithmic function. So, the only solution is $x = 6$.

Your Turn

103. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

No Solution: Both -2 and -3 are extraneous

104. $\ln(x + 1) - \ln(x - 2) = \ln x$

3.303 or $\frac{3+\sqrt{13}}{2}$

$\frac{3-\sqrt{13}}{2}$ is extraneous

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Practice 1 - Solve the Logarithmic equation

1) $\log(3x-9) = \log(2x+6)$

$x = 15$

2) $\log(-4n+7) = \log 3n$

$n = 1$

3) $\log n = \log 12$

$n = 12$

4) $\log(5x-7) = \log(3x-1)$

$x = 3$

5) $1 + \log_5 -9b = 4$

$b = \frac{-125}{9}$

6) $-7\log_4 -10r = -14$

$r = \frac{-8}{5}$

7) $4\log_{11}(r+8) = 8$

$r = 113$

8) $\log_3(x+1) - 5 = -5$

$x = 0$

9) $\log_{18}(3k^2 - 5k) = \log_{18}(-6+2k^2)$

$k = 2.3$

10) $\log_{14}(6v-1) = \log_{14}(v^2-17)$

$r = 8$

11) $\log_{19}(7-3r^2) = \log_{19}(-2r^2-6r)$

$r = -1$

12) $\log_{14}(-32-3n) = \log_{14}(n^2+9n)$

No solution

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Practice 2 - Solve the Logarithmic equation, round 3 decimal places.

1) $\log x - \log 2 = \log 17$

$x = 34$

2) $\log 8 + \log x = 1$

$x = 1.25$

3) $\log 3 + \log x = 2$

$x = 33.\overline{3}$

4) $\log x - \log 2 = 1$

$x = 20$

Practice 3 - Solve the Logarithmic equation, use fractions if necessary.

5) $\log_8 (x^2 - 1) - \log_8 3 = 1$

$x = 5, -5$

6) $\log 3x^2 - \log 3 = 2$

$x = 10, -10$

7) $\log_8 4x - \log_8 5 = \log_8 39$

$x = \frac{195}{4}$

8) $\log_7 (x+4) - \log_7 x = 3$

$x = \frac{2}{171}$

9) $\ln (5 - 2x) + \ln 9 = 4$

$x = \frac{e^4 + 45}{18}$

10) $\ln (3x - 1) + \ln 4 = \ln 15$

$x = \frac{19}{12}$

11) $\ln (10 - 2x^2) - \ln 5 = \ln 2$

$x = 0$

12) $\ln 5 - \ln (4 - 4x) = \ln 33$

$x = \frac{127}{143}$