Lesson 2.9 - Solving Logarithmic Equations

Learning Objectives - SWBAT:

1. Solve Logarithmic Equations

Solving Logarithmic Equations

To solve a logarithmic equation, you can write it in exponential form.

$\ln x = 3$	Logarithmic form
$e^{\ln x} = e^3$	Exponentiate each side.
$x = e^3$	Exponential form

This procedure is called *exponentiating* each side of an equation. It is applied after the logarithmic expression has been isolated.

Example 6 Solving Logarithmic Equations

Solve each logarithmic equation.

a. $\ln 3x = 2$ **b.** $\log_3(5x - 1) = \log_3(x + 7)$

Solution

a.

$\ln 3x = 2$	Write original equation.
$e^{\ln 3x} = e^2$	Exponentiate each side.
$3x = e^2$	Inverse Property
$x = \frac{1}{3}e^2 \approx 2.46$	Multiply each side by $\frac{1}{3}$.

The solution is $x = \frac{1}{3}e^2 \approx 2.46$. Check this in the original equation.

b.

$\log_3(5x - 1) = \log_3(x + 7)$	Write original equation.
$5x - 1 = x + 7$	One-to-One Property
$x = 2$	Solve for x.

The solution is $x = 2$. Check this in the original equation.

Your Turn

87. $\ln 4x = 2.1$

92. $\log_9(4 + x) = \log_9(2x - 1)$

Example 7 Solving a Logarithmic Equation

Solve $5 + 2\ln x = 4$.

Algebraic Solution

$5 + 2\ln x = 4$	Write original equation.
$2\ln x = -1$	Subtract 5 from each side.
$\ln x = -\frac{1}{2}$	Divide each side by 2.
$e^{\ln x} = e^{-1/2}$	Exponentiate each side.
$x = e^{-1/2}$	Inverse Property
$x \approx 0.61$	Use a calculator.

The solution is $x = e^{-1/2} \approx 0.61$. Check this in the original equation.

Your Turn

89. $-2 + 2\ln 3x = 17$

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Example 8 Solving a Logarithmic Equation

Solve $2\log_5 3x = 4$.

Solution

$2\log_5 3x = 4$	Write original equation.
$\log_5 3x = 2$	Divide each side by 2.
$5^{\log_5 3x} = 5^2$	Exponentiate each side (base 5).
$3x = 25$	Inverse Property
$x = \frac{25}{3}$	Divide each side by 3.

Your Turn

95. $7\log_4(0.6x) = 12$

96. $4\log_{10}(x - 6) = 11$

Example 9 Checking for Extraneous Solutions

Solve $\ln(x - 2) + \ln(2x - 3) = 2\ln x$.

Algebraic Solution

$\ln(x - 2) + \ln(2x - 3) = 2\ln x$	Write original equation.
$\ln[(x - 2)(2x - 3)] = \ln x^2$	Use properties of logarithms.
$\ln(2x^2 - 7x + 6) = \ln x^2$	Multiply binomials.
$2x^2 - 7x + 6 = x^2$	One-to-One Property
$x^2 - 7x + 6 = 0$	Write in general form.
$(x - 6)(x - 1) = 0$	Factor.
$x - 6 = 0 \Rightarrow x = 6$	Set 1st factor equal to 0.
$x - 1 = 0 \Rightarrow x = 1$	Set 2nd factor equal to 0.

Finally, by checking these two "solutions" in the original equation, you can conclude that $x = 1$ is not valid. This is because when $x = 1$, $\ln(x - 2) + \ln(2x - 3) = \ln(-1) + \ln(-1)$, which is invalid because -1 is not in the domain of the natural logarithmic function. So, the only solution is $x = 6$.

Your Turn

103. $\ln(x + 5) = \ln(x - 1) - \ln(x + 1)$

104. $\ln(x + 1) - \ln(x - 2) = \ln x$

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Practice 1 - Solve the Logarithmic equation

1) $\log(3x-9)=\log(2x+6)$

2) $\log(-4n+7)=\log 3n$

3) $\log n=\log 12$

4) $\log(5x-7)=\log(3x-1)$

5) $1+\log_5 -9b=4$

6) $-7\log_4 -10r=-14$

7) $4\log_{11}(r+8)=8$

8) $\log_3(x+1)-5=-5$

9) $\log_{18}(3k^2-5k)=\log_{18}(-6+2k^2)$

10) $\log_{14}(6v-1)=\log_{14}(v^2-17)$

11) $\log_{19}(7-3r^2)=\log_{19}(-2r^2-6r)$

12) $\log_{14}(-32-3n)=\log_{14}(n^2+9n)$

Lesson 2.9 - Solving Logarithmic Equations

Practice 2 - Solve the Logarithmic equation, round 3 decimal places.

1) $\log x - \log 2 = \log 17$

2) $\log 8 + \log x = 1$

3) $\log 3 + \log x = 2$

4) $\log x - \log 2 = 1$

Practice 3 - Solve the Logarithmic equation, use fractions if necessary.

5) $\log_8 (x^2 - 1) - \log_8 3 = 1$

6) $\log 3x^2 - \log 3 = 2$

7) $\log_8 4x - \log_8 5 = \log_8 39$

8) $\log_7 (x + 4) - \log_7 x = 3$

9) $\ln (5 - 2x) + \ln 9 = 4$

10) $\ln (3x - 1) + \ln 4 = \ln 15$

11) $\ln (10 - 2x^2) - \ln 5 = \ln 2$

12) $\ln 5 - \ln (4 - 4x) = \ln 33$