

## Lesson 3.10 - Bearings

Learning Objectives: SWBAT

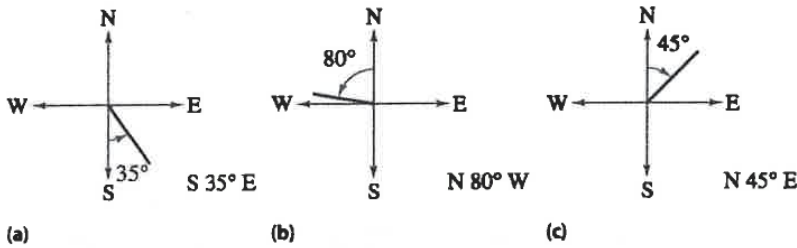
1. Sketch the angles associated with bearings in real world problems
2. Solve for distances and bearings using trigonometry

### Making a connection

- Previously, we learned the rules for solving right triangles (SOH CAH TOA) and oblique triangles (Laws of sines/cosines)
- This lesson extends these concepts to real world problems involving navigation that rely on BEARINGS

### What is a bearing?

- A bearing measures the acute angle that a path (or line of sight) makes with a fixed north or south line.
- Understanding compass directions are a big help in understanding bearings
- In the example "a" below, the bearing of **S 35° E** means **35° east of south**



- Sometimes, in air navigation, a direction may not be given. If this is the case, we assume that the angle of the bearing opens clockwise from the north. In these cases, the angle given may not always be acute.



Example: A ship leaves port at noon and heads due west at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship changes course to N 54° W, as shown in Figure 4.83. Find the ship's bearing and distance from the port of departure at 3 P.M.

AH  
interior  
∠'s

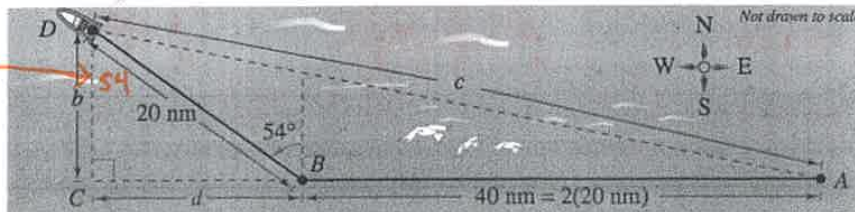


Figure 4.83

distance to port = side c

$$\begin{aligned}
 c^2 &= 20^2 + 40^2 - (2)(20)(40)\cos 144 \\
 c^2 &= 2000 - (-1294.4) \\
 c^2 &= 3294.4 \quad \boxed{c = 57.4 \text{ nm}}
 \end{aligned}$$

Bearing:  $\angle D = \frac{\sin D}{40} = \frac{\sin 144}{57.4}$

$\angle D = \sin^{-1}\left(\frac{40 \sin 144}{57.4}\right)$

$\angle D = 24.2^\circ \rightarrow \text{add } 54^\circ$

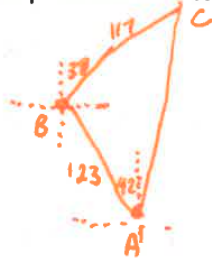
**Bearing  $571.2^\circ \text{ W}$**

### Lesson 3.10 - Bearings

Practice: Draw a diagram of each scenario. Answer any other questions indicated

1. A surveyor starts at a point A, proceeds on a bearing of  $N42^{\circ}W$  for 123' to point B. He then turns on a bearing of  $N38^{\circ}E$  and proceeds 117' to point C.

$$\begin{aligned} \angle B &= (90-38) + (90-42) \\ &= 52 + 48 \\ &= 100^{\circ} \end{aligned}$$



$$AC^2 = 117^2 + 123^2 - 2(117)(123)\cos 100$$

$$AC^2 = 13689 + 15129 + 4794.8$$

$$AC^2 = 33612.8$$

$$AC = 183.3'$$

$$\angle A = \frac{\sin A}{117} = \frac{\sin 100}{183.3} \rightarrow \angle A = 38.9^{\circ}$$

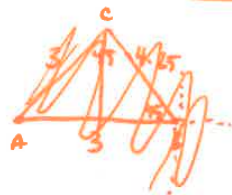
$$\angle C = 180 - 100 - 38.9 = 41.1^{\circ}$$

- ~~2. A ship heads on a bearing of  $230^{\circ}$  for 1,270 nautical miles, then turns on a bearing of  $290^{\circ}$ .~~

- ~~3. A robotic submersible vessel travels on a bearing of  $S18^{\circ}E$  for 2.7 miles, then dives beneath an ice shelf, turns on a bearing of  $N27^{\circ}E$  and travels for 5.2 miles. What angle did the vessel make when it turned underneath the ice shelf?~~

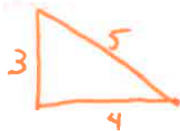
4. A surveyor starts at point A. She travels due East for 3 miles, to point B. Then she turns on a bearing of  $N45^{\circ}W$  and measures 4.25 miles to point C. She then turns due South and walks 3 miles back to point A. What angle did the surveyor make at point B? What shape did the surveyor measure out?

$$\begin{aligned} \angle B &= 45^{\circ} \\ \text{Bearing: } &S45E \end{aligned}$$



→ Isosceles right  $\Delta$

5. A search party walks due South 3 miles, then turns and walks due East for 4 miles. What is the bearing back to their starting point? How far are they from their starting point?



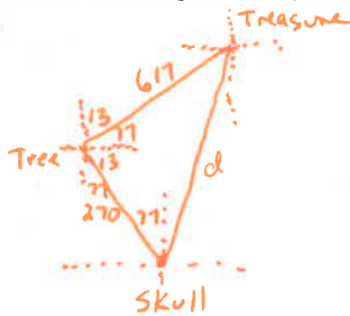
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6. You found a treasure map! The treasure map reads, "From the baboon's skull, mark 270 paces on a bearing  $N77^\circ W$ , till ye find the hangin' tree. From the hangin' tree, mark yer bearing  $N13^\circ E$  and count 617 paces. Thar ye'll find the treasure, guarded by a nest o rattlers and a dead man's curse!" How far is the treasure from the baboon's skull, and in what direction? What is a baboon's skull doing there, anyway?

$\angle T$  (Treasure)  
 $\tan^{-1} \left( \frac{270}{617} \right)$   
 $\angle T = 23.6^\circ$

Bearing  
 $S66.4^\circ W$



Distance from treasure to skull

$$270^2 + 617^2 = d^2$$

$$\boxed{673.5 \text{ paces} = d}$$

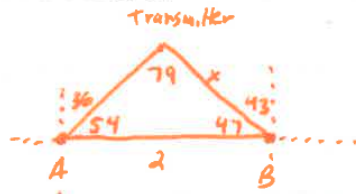
**Problem 1:** A ship travels 70 km on a bearing of  $27^\circ$ , and then travels on a bearing of  $147^\circ$  for 180 km. Find the distance of the end of the trip from the starting point.

**Problem 2:** Two lighthouses are located on a north-south line. From lighthouse A the bearing of a ship 3742 m away is  $129^\circ 43'$ . From lighthouse B the bearing of the ship is  $39^\circ 43'$ . Find the distance between the lighthouses.

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**Problem 3:** Radio direction finders are set up at points A and B, which are 2.00 mi. apart on an east-west line. From A it is found that the bearing of the signal from a radio transmitter is N 36° 20' E, while from B the bearing of the same signal is N 43° 40' W. Find the distance of the transmitter from B.



$$\frac{\sin 79}{2} = \frac{\sin 54}{x}$$

$$x = \frac{2 \sin 54}{\sin 79}$$

$$x = 1.65 \text{ mi}$$

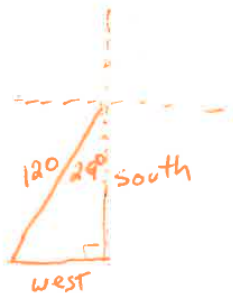
**Problem 4:** Radar stations A and B are on an east-west line, 3.7 miles apart. Station A detects a plane at C, on a bearing of 61°. Station B detects the plane at a bearing of 321°. Find the distance from A to C.

**33. Navigation** A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots. How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.? (6 hours at 20 knots/hour)

South  
 $\cos 29 = \frac{\text{South}}{120}$

$$\text{South} = 120 \cos 29$$

$$\text{South} = 104.95 \text{ miles}$$



$$\sin 29 = \frac{\text{West}}{120}$$

$$\text{West} = 120 \sin 29$$

$$\text{West} = 58.18 \text{ miles}$$

$$\text{Check: } 104.95^2 + 58.18^2 = 120^2 \checkmark$$

## Lesson 3.10 - Bearings

**Practice:** Draw a diagram of each scenario. Answer any other questions indicated

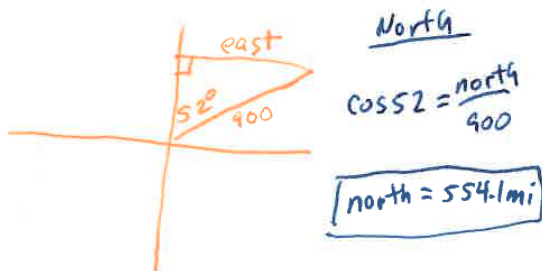
34. **Navigation** An airplane flying at 600 miles per hour has a bearing of  $52^\circ$ . After flying for 1.5 hours, how far north and how far east has the plane traveled from its point of departure?

$$600 \times 1.5 = 900 \text{ miles}$$

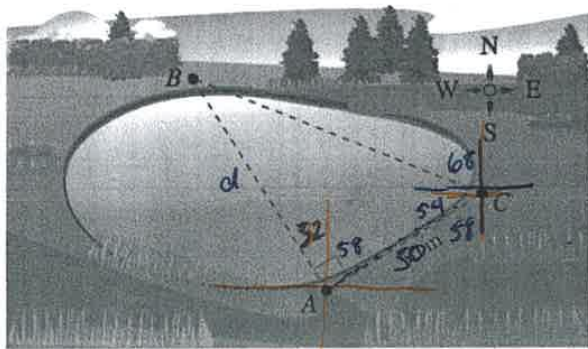
$$\sin 52 = \frac{\text{east}}{900}$$

$$\text{east} = 900 \sin 52$$

$$\text{east} = 709.2 \text{ miles}$$



35. **Surveying** A surveyor wants to find the distance across a pond (see figure). The bearing from  $A$  to  $B$  is  $N 32^\circ W$ . The surveyor walks 50 meters from  $A$ , and at the point  $C$  the bearing to  $B$  is  $N 68^\circ W$ . Find (a) the bearing from  $A$  to  $C$  and (b) the distance from  $A$  to  $B$ .



Bearing from A to C

$$90 - 32 = 58^\circ$$

$$= \boxed{N 58^\circ E}$$

Distance from A to B

$$\angle C = 180 - 68 - 58 = 54^\circ$$

$$\tan 54 = \frac{d}{50}$$

$$d = 50 (\tan 54) = 68.82 \text{ m}$$

36. **Location of a Fire** Two fire towers are 30 kilometers apart, where tower  $A$  is due west of tower  $B$ . A fire is spotted from the towers, and the bearings from  $A$  and  $B$  are  $E 14^\circ N$  and  $W 34^\circ N$ , respectively (see figure). Find the distance  $d$  of the fire from the line segment  $AB$ .



Find  $\angle C$

$$\angle ACB = 180 - 14 - 34 = 132^\circ$$

$$\frac{\sin 34}{AC} = \frac{\sin 132}{30}$$

$$AC = \frac{30 \sin 34}{\sin 132} = 22.57 \text{ km}$$

Find  $d$

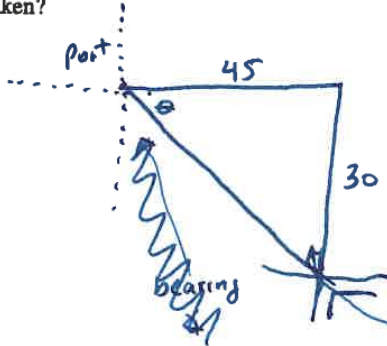
$$\sin 14 = \frac{d}{22.57}$$

$$d = 5.46 \text{ km}$$

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37. **Navigation** A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?



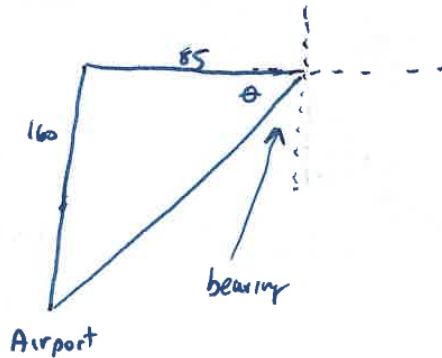
$$\tan \theta = \frac{30}{45}$$

$$\theta = \tan^{-1}\left(\frac{30}{45}\right) = 33.7^\circ$$

$$\text{Bearing } \theta = 90 - 33.7 = 56.3^\circ$$

$$\text{Bearing: } \boxed{S 56.3^\circ W}$$

38. **Navigation** A plane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should be taken?



$$\tan \theta = \frac{160}{85}$$

$$\theta = \tan^{-1}\left(\frac{160}{85}\right) = 62.02^\circ$$

$$\text{bearing} = 90 - 62.02 = 27.98^\circ$$

$$\text{bearing: } S 27.98^\circ W$$

180. **Distance** From city A to city B, a plane flies 650 miles at a bearing of  $48^\circ$ . From city B to city C, the plane flies 810 miles at a bearing of  $115^\circ$ . Find the distance from A to C and the bearing from A to C.

