Lesson 3.~~12~~ - Vector Basics
11

Learning Objectives: SWBAT

1. Represent vectors as directed line segments
2. Write the component forms and determine the magnitude of vectors

Background

Many quantities in geometry and physics, such as area, time, and temperature, can be represented by a single real number. Other quantities, such as force and velocity, involve both *magnitude* and *direction* and cannot be completely characterized by a single real number. To represent such a quantity, you can use a **directed line segment**, as shown in Figure 6.15. The directed line segment $\overrightarrow{PQ}$ has **initial point** P and **terminal point** Q. Its **magnitude**, or **length**, is denoted by $\|\overrightarrow{PQ}\|$ and can be found by using the Distance Formula.

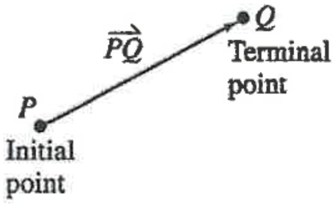

Figure 6.15

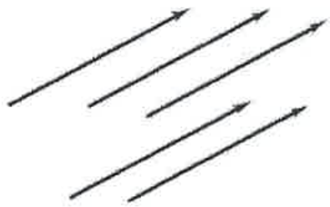

Figure 6.16

Two directed line segments that have the same magnitude and direction are *equivalent*. For example, the directed line segments in Figure 6.16 are all equivalent. The set of all directed line segments that are equivalent to a given directed line segment $\overrightarrow{PQ}$ is a **vector v in the plane,** written $\mathbf{v} = \overrightarrow{PQ}$. Vectors are denoted by lowercase, boldface letters such as **u**, **v**, and **w**.

The basics of vectors

Let **u** be represented by the directed line segment from $P = (0, 0)$ to $Q = (3, 2)$, and let **v** be represented by the directed line segment from $R = (1, 2)$ to $S = (4, 4)$, as shown in Figure 6.17. Show that $\mathbf{u} = \mathbf{v}$.

Solution

From the Distance Formula, it follows that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the *same magnitude*.

$$\|\overrightarrow{PQ}\| = \sqrt{(3-0)^2 + (2-0)^2} = \sqrt{13}$$

$$\|\overrightarrow{RS}\| = \sqrt{(4-1)^2 + (4-2)^2} = \sqrt{13}$$

Moreover, both line segments have the *same direction*, because they are both directed toward the upper right on lines having the same slope.

$$\text{Slope of } \overrightarrow{PQ} = \frac{2-0}{3-0} = \frac{2}{3}$$

$$\text{Slope of } \overrightarrow{RS} = \frac{4-2}{4-1} = \frac{2}{3}$$

So, $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the same magnitude and direction, and it follows that $\mathbf{u} = \mathbf{v}$

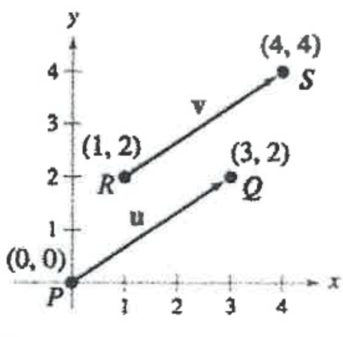

Figure 6.17

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Component Form

The directed line segment whose initial point is the origin is often the most convenient representative of a set of equivalent directed line segments. This representative of the vector **v** is in **standard position.**

A vector whose initial point is at the origin (0, 0) can be uniquely represented by the coordinates of its terminal point (v_1, v_2). This is the **component form of a vector v,** written as

$$\mathbf{v} = \langle v_1, v_2 \rangle.$$

- Another way to look at component form is that it quantifies the direction of the vector in terms of its slope: **v = < run, rise >**

Example:

Find the component form and magnitude of the vector **v** that has initial point (4, −7) and terminal point (−1, 5).

Solution

Let $P = (4, -7) = (p_1, p_2)$ and $Q = (-1, 5) = (q_1, q_2)$, as shown in Figure 6.18. Then, the components of $\mathbf{v} = \langle v_1, v_2 \rangle$ are

$$v_1 = q_1 - p_1 = -1 - 4 = -5$$

$$v_2 = q_2 - p_2 = 5 - (-7) = 12.$$

So, $\mathbf{v} = \langle -5, 12 \rangle$ and the magnitude of **v** is

$$\|\mathbf{v}\| = \sqrt{(-5)^2 + 12^2} = \sqrt{169} = 13.$$

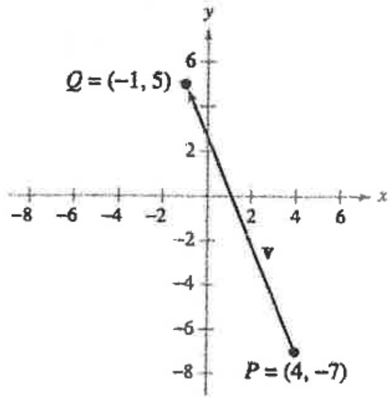


Figure 6.18

Your Turn - Find the component form and magnitude of the following

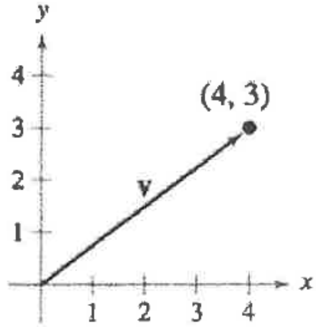
1) $\overrightarrow{RS}$ where $R = (-3, 9)$ $S = (8, -1)$

$\langle 11, -10 \rangle$, $\|V\| = \sqrt{221}$ or 14.87

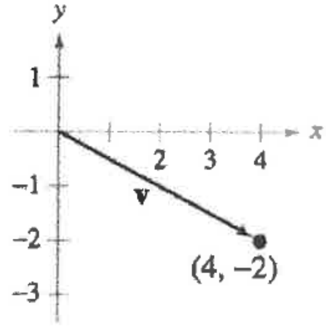
2) $\overrightarrow{PQ}$ where $P = (-10, 5)$ $Q = (-9, -10)$

$\langle 1, -15 \rangle$, $\|v\| = \sqrt{226}$ or 15.1

3.



4.



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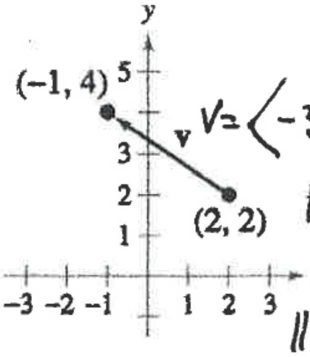
Practice - Find the component form and magnitude of the following

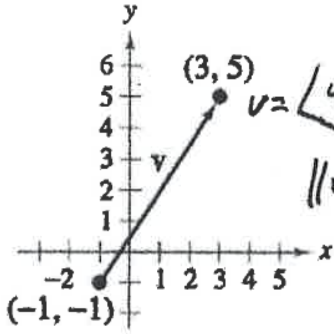
5.



$V = \langle -3, 2 \rangle$

$\|v\| = \sqrt{13}$ or 3.61

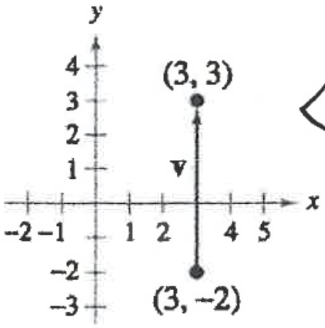
6.



$v = \langle 4, 6 \rangle$

$\|v\| = 2\sqrt{13}$ or 7.21

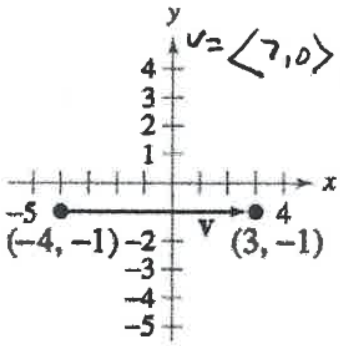
7.



$\langle 0, 5 \rangle$

$\|v\| = 5$

8.



$v = \langle 7, 0 \rangle$, $\|v\| = 7$

	Initial Point	*Terminal Point*		*Initial Point*	*Terminal Point*
9.	$\left(\frac{2}{5}, 1\right)$	$\left(1, \frac{2}{5}\right)$	**10.**	$\left(\frac{7}{2}, 0\right)$	$\left(0, -\frac{7}{2}\right)$

$V = \left\langle \frac{3}{5}, -\frac{3}{5} \right\rangle$

$\|v\| = \frac{3\sqrt{2}}{5}$

$V = \left\langle -\frac{7}{2}, -\frac{7}{2} \right\rangle$

$\|V\| = \frac{7\sqrt{2}}{2}$

11.	$\left(-\frac{2}{3}, -1\right)$	$\left(\frac{1}{2}, \frac{4}{5}\right)$	**12.**	$\left(\frac{5}{2}, -2\right)$	$\left(1, \frac{2}{5}\right)$

$V = \left\langle \frac{7}{6}, \frac{9}{5} \right\rangle$ $\|v\| = \frac{\sqrt{4141}}{30} = 2.145$

$V = \left\langle -\frac{3}{2}, \frac{12}{5} \right\rangle$

$\|v\| = \frac{3\sqrt{89}}{10}$ or 2.83