Lesson 3.~~13~~ 12 - Vector Operations

Learning Objectives: SWBAT

1. Apply addition, subtraction and scalar multiplication on vectors
2. Find the unit vector in the direction of a vector (v)

Addition and Scalar Multiplication Examples: Notice how each example relates to the corresponding graph

Let $\mathbf{v} = \langle -2, 5 \rangle$ and $\mathbf{w} = \langle 3, 4 \rangle$, and find each of the following vectors.

a. $2\mathbf{v}$ **b.** $\mathbf{w} - \mathbf{v}$ **c.** $\mathbf{v} + 2\mathbf{w}$ **d.** $2\mathbf{v} - 3\mathbf{w}$

Solution

a. Because $\mathbf{v} = \langle -2, 5 \rangle$, you have

$$\begin{aligned} 2\mathbf{v} &= 2\langle -2, 5 \rangle \\ &= \langle 2(-2), 2(5) \rangle \\ &= \langle -4, 10 \rangle. \end{aligned}$$

A sketch of $2\mathbf{v}$ is shown in Figure 6.22.

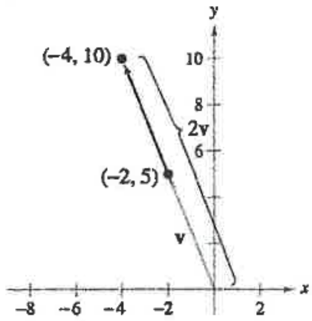

Figure 6.22

b. The difference of $\mathbf{w}$ and $\mathbf{v}$ is

$$\begin{aligned} \mathbf{w} - \mathbf{v} &= \langle 3 - (-2), 4 - 5 \rangle \\ &= \langle 5, -1 \rangle. \end{aligned}$$

A sketch of $\mathbf{w} - \mathbf{v}$ is shown in Figure 6.23. Note that the figure shows the vector difference $\mathbf{w} - \mathbf{v}$ as the sum $\mathbf{w} + (-\mathbf{v})$.

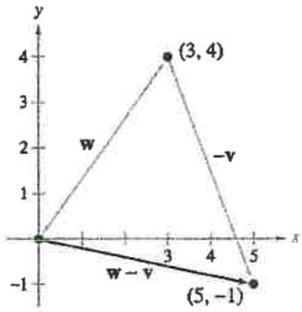

Figure 6.23

c. The sum of $\mathbf{v}$ and $2\mathbf{w}$ is

$$\begin{aligned} \mathbf{v} + 2\mathbf{w} &= \langle -2, 5 \rangle + 2\langle 3, 4 \rangle \\ &= \langle -2, 5 \rangle + \langle 2(3), 2(4) \rangle \\ &= \langle -2, 5 \rangle + \langle 6, 8 \rangle \\ &= \langle -2 + 6, 5 + 8 \rangle \\ &= \langle 4, 13 \rangle. \end{aligned}$$

A sketch of $\mathbf{v} + 2\mathbf{w}$ is shown in Figure 6.24.

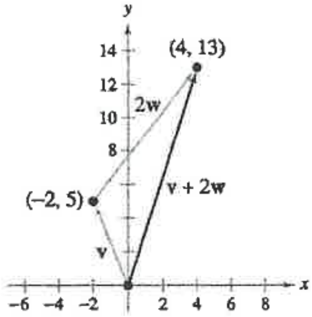

Figure 6.24

d. The difference of $2\mathbf{v}$ and $3\mathbf{w}$ is

$$\begin{aligned} 2\mathbf{v} - 3\mathbf{w} &= 2\langle -2, 5 \rangle - 3\langle 3, 4 \rangle \\ &= \langle 2(-2), 2(5) \rangle - \langle 3(3), 3(4) \rangle \\ &= \langle -4, 10 \rangle - \langle 9, 12 \rangle \\ &= \langle -4 - 9, 10 - 12 \rangle \\ &= \langle -13, -2 \rangle. \end{aligned}$$

A sketch of $2\mathbf{v} - 3\mathbf{w}$ is shown in Figure 6.25. Note that the figure shows the vector difference $2\mathbf{v} - 3\mathbf{w}$ as the sum $2\mathbf{v} + (-3\mathbf{w})$.

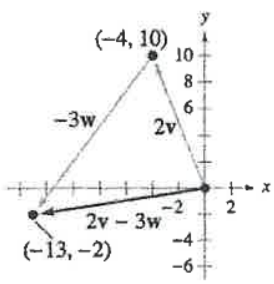

Figure 6.25

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Practice In Exercises 25–30, find (a) $\mathbf{u} + \mathbf{v}$, (b) $\mathbf{u} - \mathbf{v}$, (c) $2\mathbf{u} - 3\mathbf{v}$, and (d) $\mathbf{v} + 4\mathbf{u}$.

25. $\mathbf{u} = \langle 4, 2\rangle$, $\mathbf{v} = \langle 7, 1\rangle$

(a) $u+v = \langle 11, 3\rangle$

(b) $u-v = \langle -3, 1\rangle$

(c) $2u-3v = \langle 8, 4\rangle$

(d) $v+4u = \langle 23, 9\rangle$

26. $\mathbf{u} = \langle 5, 3\rangle$, $\mathbf{v} = \langle -4, 0\rangle$

(a) $u+v = \langle 1, 3\rangle$

(b) $u-v = \langle 9, 3\rangle$

(c) $2u-3v = \langle 22, 6\rangle$

(d) $v+4u = \langle 16, 12\rangle$

27. $\mathbf{u} = \langle -6, -8\rangle$, $\mathbf{v} = \langle 2, 4\rangle$

(a) $u+v = \langle -4, -4\rangle$

(b) $u-v = \langle -8, -12\rangle$

(c) $2u-3v = \langle -18, -28\rangle$

(d) $v+4u = \langle -22, -28\rangle$

28. $\mathbf{u} = \langle 0, -5\rangle$, $\mathbf{v} = \langle -3, 9\rangle$

(a) $u+v = \langle -3, 4\rangle$

(b) $u-v = \langle 3, -14\rangle$

(c) $2u-3u = \langle 9, -37\rangle$

(d) $v+4u = \langle -3, -11\rangle$

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Practice

Find the component form of the resultant vector.

1) $\vec{u} = \langle 20, -21 \rangle$
Find: $-3\vec{u}$

$\langle -60, 63 \rangle$

2) Given: $P = (0, -4)$ $Q = (-1, 9)$
Find: $8\overrightarrow{PQ}$

$\langle -8, 104 \rangle$

3) $\vec{u} = \langle 3, 3 \rangle$
$\vec{v} = \langle 11, 8 \rangle$
Find: $\vec{u} + \vec{v}$

$\langle 14, 11 \rangle$

4) Given: $P = (-7, -6)$ $Q = (6, 10)$
$R = (-3, -9)$ $S = (-3, 7)$
Find: $\overrightarrow{PQ} + \overrightarrow{RS}$

$\langle 13, 32 \rangle$

5) $\vec{f} = \langle 12, 2 \rangle$
$\vec{v} = \langle 2, 4 \rangle$
Find: $4\vec{f} - 6\vec{v}$

$\langle 36, -16 \rangle$

6) Given: $T = (-3, 8)$ $X = (3, 10)$
$Y = (-4, -7)$ $Z = (-8, -10)$
Find: $4\overrightarrow{TX} + \overrightarrow{YZ}$

$\langle 20, 5 \rangle$

7) Given: $A = (9, 3)$ $B = (-9, -9)$
$C = (-4, 10)$ $D = (5, 5)$
Find: $-\overrightarrow{AB} + \overrightarrow{CD}$

$\left\langle \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

8) Given: $A = (-6, 0)$ $B = (-4, -1)$
$C = (7, 5)$ $D = (4, 4)$
Find: $7\overrightarrow{AB} - 5\overrightarrow{CD}$

$\langle -6, -5 \rangle$

Lesson 3.13 - Vector Operations

Unit Vectors

In many applications of vectors, it is useful to find a unit vector that has the same direction as a given nonzero vector **v**. To do this, you can divide **v** by its length to obtain

$$\mathbf{u} = \text{unit vector} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{\|\mathbf{v}\|}\right)\mathbf{v}. \qquad \text{Unit vector in direction of } \mathbf{v}$$

Note that **u** is a scalar multiple of **v**. The vector **u** has a magnitude of 1 and the same direction as **v**. The vector **u** is called a **unit vector in the direction of v.**

Example: Find a unit vector in the direction of $\mathbf{v} = \langle -2, 5 \rangle$ and verify that the result has a magnitude of 1.

Solution

The unit vector in the direction of **v** is

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{\langle -2, 5 \rangle}{\sqrt{(-2)^2 + (5)^2}}$$

$$= \frac{1}{\sqrt{29}}\langle -2, 5 \rangle$$

$$= \left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle = \left\langle \frac{-2\sqrt{29}}{29}, \frac{5\sqrt{29}}{29} \right\rangle.$$

This vector has a magnitude of 1 because

$$\sqrt{\left(\frac{-2\sqrt{29}}{29}\right)^2 + \left(\frac{5\sqrt{29}}{29}\right)^2} = \sqrt{\frac{116}{841} + \frac{725}{841}} = \sqrt{\frac{841}{841}} = 1.$$

Practice **In Exercises 35–44, find a unit vector in the direction of the given vector.**

35. $\mathbf{u} = \langle 6, 0 \rangle$

$\|u\| = 6 \quad \frac{1}{6}\langle 6, 0 \rangle = \langle 1, 0 \rangle$

36. $\mathbf{u} = \langle 0, -2 \rangle$

$\frac{1}{2}\langle 0, -2 \rangle = \langle 0, -1 \rangle$

37. $\mathbf{v} = \langle -1, 1 \rangle$

$\|v\| = \sqrt{2} \quad \frac{1}{\sqrt{2}}\langle -1, 1 \rangle = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

38. $\mathbf{v} = \langle 3, -4 \rangle$

$\|v\| = 5 \quad \frac{1}{5}\langle 3, -4 \rangle = \left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

39. $\mathbf{v} = \langle -24, -7 \rangle$

$\|v\| = 25 \quad \frac{1}{25}\langle -24, -7 \rangle = \left\langle \frac{-24}{25}, \frac{-7}{25} \right\rangle$

40. $\mathbf{v} = \langle 8, -20 \rangle$

$\|v\| = \sqrt{29} \quad \frac{1}{\sqrt{29}}\langle 8, -20 \rangle = \left\langle \frac{2\sqrt{29}}{29}, \frac{-5\sqrt{29}}{29} \right\rangle$