13

Lesson 3.14 - Linear combinations of vectors

Learning Objectives: SWBAT

1. Write a linear combination of two unit vector "i" and "j"
2. Perform operations on linear combinations of vectors

Background

The unit vectors $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$ are called the **standard unit vectors** and are denoted by

$$\mathbf{i} = \langle 1, 0\rangle \qquad \text{and} \qquad \mathbf{j} = \langle 0, 1\rangle$$

as shown in Figure 6.26. (Note that the lowercase letter **i** is written in boldface to distinguish it from the imaginary number $i = \sqrt{-1}$.) These vectors can be used to represent any vector $\mathbf{v} = \langle v_1, v_2\rangle$ as follows.

$$\begin{aligned}\mathbf{v} &= \langle v_1, v_2\rangle \\ &= v_1\langle 1, 0\rangle + v_2\langle 0, 1\rangle \\ &= v_1\mathbf{i} + v_2\mathbf{j}\end{aligned}$$

The scalars v_1 and v_2 are called the **horizontal and vertical components of v**, respectively. The vector sum

$$v_1\mathbf{i} + v_2\mathbf{j}$$

is called a **linear combination** of the vectors **i** and **j**. Any vector in the plane can be written as a linear combination of the standard unit vectors **i** and **j**.

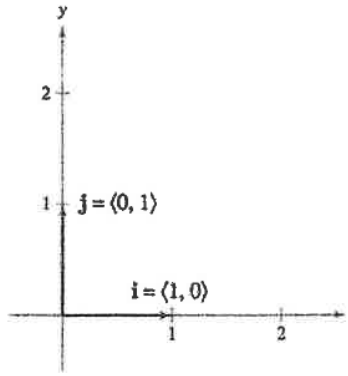

Figure 6.26

Example - Writing linear combinations of vectors

Let **u** be the vector with initial point $(2, -5)$ and terminal point $(-1, 3)$. Write **u** as a linear combination of the standard unit vectors **i** and **j**.

Solution

Begin by writing the component form of the vector **u**.

$$\begin{aligned}\mathbf{u} &= \langle -1 - 2, 3 - (-5)\rangle \\ &= \langle -3, 8\rangle \\ &= -3\mathbf{i} + 8\mathbf{j}\end{aligned}$$

This result is shown graphically in Figure 6.27.

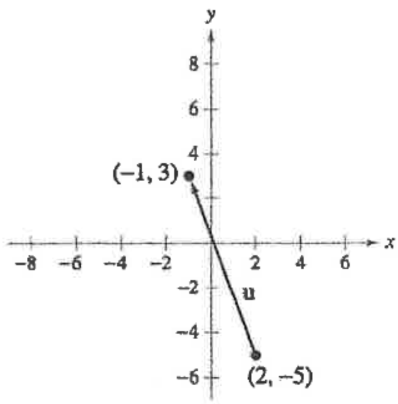

Figure 6.27

Practice **In Exercises 51–54, the initial and terminal points of a vector are given. Write the vector as a linear combination of the standard unit vectors i and j.**

	Initial Point	*Terminal Point*	
51.	$(-3, 1)$	$(4, 5)$	$\to \langle 4-(-3), 5-1\rangle = \langle 7, 4\rangle = \boxed{7i + 4j}$
52.	$(0, -2)$	$(3, 6)$	$\to$ $= \boxed{3i + 8j}$
53.	$(-1, -5)$	$(2, 3)$	$\to$ $= \boxed{3i + 8j}$
54.	$(-6, 4)$	$(0, 1)$	$\to$ $= \boxed{6i - 3j}$

13

Lesson 3.~~14~~ - Linear combinations of vectors

Example - Operating with Linear Combinations

Let $\mathbf{u} = -3\mathbf{i} + 8\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$. Find $2\mathbf{u} - 3\mathbf{v}$.

Solution

You could solve this problem by converting **u** and **v** to component form. This, however, is not necessary. It is just as easy to perform the operations in unit vector form.

$$\begin{aligned} 2\mathbf{u} - 3\mathbf{v} &= 2(-3\mathbf{i} + 8\mathbf{j}) - 3(2\mathbf{i} - \mathbf{j}) \\ &= -6\mathbf{i} + 16\mathbf{j} - 6\mathbf{i} + 3\mathbf{j} \\ &= -12\mathbf{i} + 19\mathbf{j} \longrightarrow \text{Component form: } \langle -12, 19 \rangle \end{aligned}$$

Practice: Let $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$. Perform the operation given to determine the combined vector. Write your answer as both a linear combination and in component form

55. $\mathbf{v} = \frac{3}{2}\mathbf{u}$

$\frac{3}{2}(2i - j) = \boxed{3i - \frac{3}{2}j}$

$= \boxed{\langle 3, -\frac{3}{2} \rangle}$

56. $\mathbf{v} = \frac{2}{3}\mathbf{w}$

$= \boxed{\frac{2}{3}i + \frac{4}{3}j}$

$= \boxed{\langle \frac{2}{3}, \frac{4}{3} \rangle}$

57. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$

$\boxed{4i + 3j}$

$\boxed{\langle 4, 3 \rangle}$

58. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

$\boxed{-i + 3j}$

$\boxed{\langle -1, 3 \rangle}$

59. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

$\boxed{\frac{7}{2}i - \frac{1}{2}j}$

$\boxed{\langle \frac{7}{2}, -\frac{1}{2} \rangle}$

60. $\mathbf{v} = 2\mathbf{u} - 2\mathbf{w}$

$\boxed{2i - 6j}$

$\boxed{\langle 2, -6 \rangle}$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$.

11. $4\mathbf{h} - \mathbf{g}$

$4\langle -6, 2 \rangle - \langle -3, -5 \rangle$

$= \langle -24, 8 \rangle - \langle -3, 5 \rangle$

$= \boxed{\langle -21, -13 \rangle = -21i - 13j}$

12. $\mathbf{f} + 2\mathbf{h}$

$\boxed{\langle -4, 4 \rangle, -4i + 4j}$

13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$

$\boxed{\langle -55, -13 \rangle \quad -55i - 13j}$

14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$

$\boxed{\langle 31, -11 \rangle \quad 31i - 11j}$

15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$

$\langle 8, 0 \rangle + (-2)\langle -3, -5 \rangle + (-2)\langle -6, 2 \rangle$

$\langle 8, 0 \rangle + \langle 6, 10 \rangle + \langle 12, -4 \rangle$

$= \boxed{\langle 26, 6 \rangle = 26i + 6j}$

16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$

$\boxed{\langle -53, -23 \rangle \quad -53i - 23j}$