Lesson 3.13 - Linear combinations of vectors

Learning Objectives: SWBAT

1. Write a linear combination of two unit vector "i" and "j"
2. Perform operations on linear combinations of vectors

Background

The unit vectors $\langle 1, 0\rangle$ and $\langle 0, 1\rangle$ are called the **standard unit vectors** and are denoted by

$$\mathbf{i} = \langle 1, 0\rangle \quad \text{and} \quad \mathbf{j} = \langle 0, 1\rangle$$

as shown in Figure 6.26. (Note that the lowercase letter **i** is written in boldface to distinguish it from the imaginary number $i = \sqrt{-1}$.) These vectors can be used to represent any vector $\mathbf{v} = \langle v_1, v_2\rangle$ as follows.

$$\begin{aligned}\mathbf{v} &= \langle v_1, v_2\rangle \\ &= v_1\langle 1, 0\rangle + v_2\langle 0, 1\rangle \\ &= v_1\mathbf{i} + v_2\mathbf{j}\end{aligned}$$

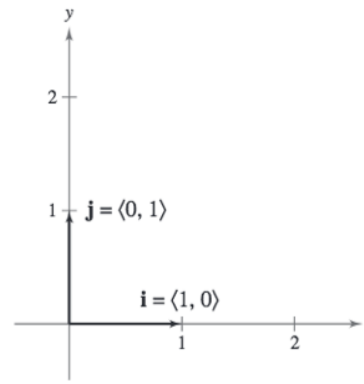

Figure 6.26

The scalars v_1 and v_2 are called the **horizontal and vertical components of v,** respectively. The vector sum

$$v_1\mathbf{i} + v_2\mathbf{j}$$

is called a **linear combination** of the vectors **i** and **j**. Any vector in the plane can be written as a linear combination of the standard unit vectors **i** and **j**.

Example - Writing linear combinations of vectors

Let **u** be the vector with initial point $(2, -5)$ and terminal point $(-1, 3)$. Write **u** as a linear combination of the standard unit vectors **i** and **j**.

Solution

Begin by writing the component form of the vector **u**.

$$\begin{aligned}\mathbf{u} &= \langle -1 - 2, 3 - (-5)\rangle \\ &= \langle -3, 8\rangle \\ &= -3\mathbf{i} + 8\mathbf{j}\end{aligned}$$

This result is shown graphically in Figure 6.27.

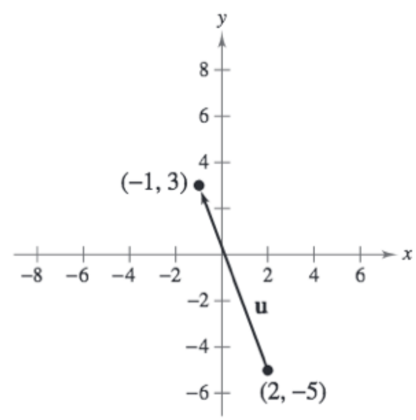

Figure 6.27

Practice

In Exercises 51–54, the initial and terminal points of a vector are given. Write the vector as a linear combination of the standard unit vectors i and j.

	Initial Point	*Terminal Point*
51.	$(-3, 1)$	$(4, 5)$
52.	$(0, -2)$	$(3, 6)$
53.	$(-1, -5)$	$(2, 3)$
54.	$(-6, 4)$	$(0, 1)$

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Example - Operating with Linear Combinations

Let $\mathbf{u} = -3\mathbf{i} + 8\mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} - \mathbf{j}$. Find $2\mathbf{u} - 3\mathbf{v}$.

Solution

You could solve this problem by converting **u** and **v** to component form. This, however, is not necessary. It is just as easy to perform the operations in unit vector form.

$$\begin{aligned} 2\mathbf{u} - 3\mathbf{v} &= 2(-3\mathbf{i} + 8\mathbf{j}) - 3(2\mathbf{i} - \mathbf{j}) \\ &= -6\mathbf{i} + 16\mathbf{j} - 6\mathbf{i} + 3\mathbf{j} \\ &= -12\mathbf{i} + 19\mathbf{j} \end{aligned}$$

$\longrightarrow$ Component form: $\langle -12, 19 \rangle$

Practice: Let $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{w} = \mathbf{i} + 2\mathbf{j}$. Perform the operation given to determine the combined vector. Write your answer as both a linear combination and in component form

55. $\mathbf{v} = \frac{3}{2}\mathbf{u}$

56. $\mathbf{v} = \frac{2}{3}\mathbf{w}$

57. $\mathbf{v} = \mathbf{u} + 2\mathbf{w}$

58. $\mathbf{v} = -\mathbf{u} + \mathbf{w}$

59. $\mathbf{v} = \frac{1}{2}(3\mathbf{u} + \mathbf{w})$

60. $\mathbf{v} = 2\mathbf{u} - 2\mathbf{w}$

Find each of the following for $\mathbf{f} = \langle 8, 0 \rangle$, $\mathbf{g} = \langle -3, -5 \rangle$, and $\mathbf{h} = \langle -6, 2 \rangle$.

11. $4\mathbf{h} - \mathbf{g}$

12. $\mathbf{f} + 2\mathbf{h}$

13. $3\mathbf{g} - 5\mathbf{f} + \mathbf{h}$

14. $2\mathbf{f} + \mathbf{g} - 3\mathbf{h}$

15. $\mathbf{f} - 2\mathbf{g} - 2\mathbf{h}$

16. $\mathbf{h} - 4\mathbf{f} + 5\mathbf{g}$