Lesson 3.15 - Direction Angles

Learning Objectives: SWBAT
1. Determine the direction angle of a given vector
2. Perform operations on linear combinations of vectors

<u>What is a direction Angle?</u>
- A direction angle is an angle that is created by a vector
- Direction angles are always measured counterclockwise from the positive side of the x axis to the terminal point of the vector
- The components of the vector determine the "run" (x coordinate) and rise (y coordinate) of the vector. These coordinates are the legs of a right triangle and are "connected" by the direction angle
- If we use the direction angle as a reference angle of the right triangle, then in order to determine the measure of the angle, we can use the Tan^{-1} function

<u>Examples</u> - How to determine the direction angle of a vector

Find the direction angle of each vector.

a. $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$ **b.** $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

Solution

a. The direction angle is

$$\tan\theta = \frac{b}{a} = \frac{3}{3} = 1. \qquad \tan^{-1}(1) = 45^\circ$$

So, $\theta = 45^\circ$, as shown in Figure 6.29.

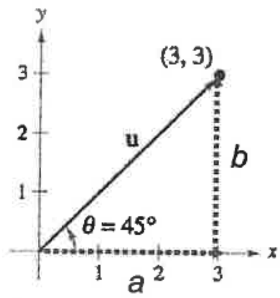


Figure 6.29

b. The direction angle is

$$\tan\theta = \frac{b}{a} = \frac{-4}{3}.$$

Moreover, because $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ lies in Quadrant IV, θ lies in Quadrant IV and its reference angle is

$$\theta' = \left|\tan^{-1}\left(-\frac{4}{3}\right)\right| \approx |-53.13^\circ| = 53.13^\circ.$$

So, it follows that $\theta \approx 360^\circ - 53.13^\circ = 306.87^\circ$, as shown in Figure 6.30.

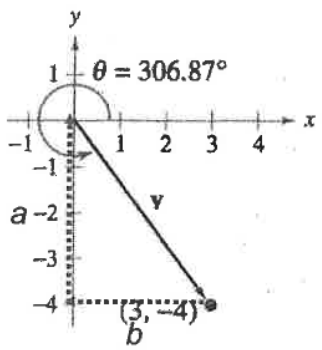


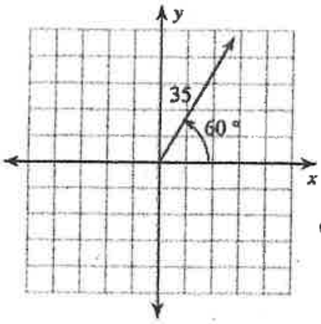
Figure 6.30

Lesson 3.~~15~~ 14 - Direction Angles

Practice

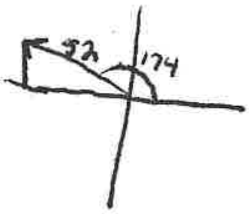
Write each vector in component form.

3)



$\left\langle \frac{35}{2}, \frac{35\sqrt{3}}{2} \right\rangle$

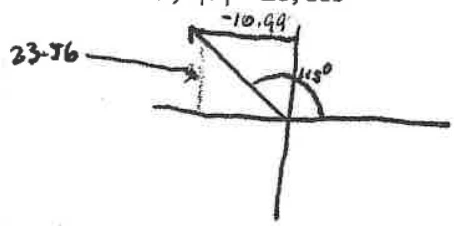
4) $|\vec{k}| = 52, 174°$



$\langle -51.72, 5.44 \rangle$

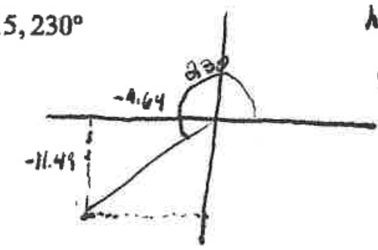
Draw a diagram to illustrate the horizontal and vertical components of the vector. Then find the magnitude of each component.

5) $|\vec{t}| = 26, 115°$



horizontal = -10.99
vertical = 23.56

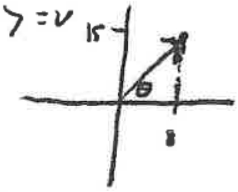
6) $|\vec{a}| = 15, 230°$



horizontal = -9.64
vertical = -11.49

Find the magnitude and direction angle for each vector.

7) $8\vec{i} + 15\vec{j}$ $\langle 8, 15 \rangle = v$



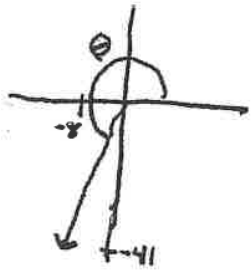
$\|v\| = 17$

$\theta = 61.93$

8) $\vec{r} = \langle -8, -41 \rangle$

$\|r\| = 41.77$

$\theta = 258.96$



Find the component form, magnitude, and direction angle for the given vector

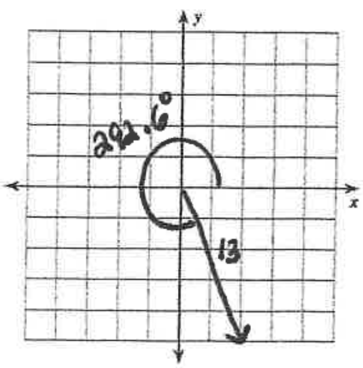
9) $\overrightarrow{CD}$ where $C = (6, -3)$ $D = (-6, -9)$ $\langle -12, -6 \rangle$

$\theta = 206.57$

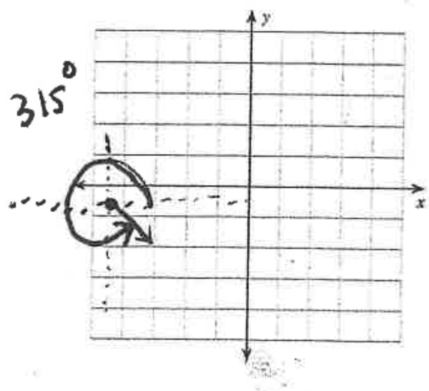
$\|CD\| = 6\sqrt{5}$ or 13.42

Sketch a graph of each vector then find the magnitude and direction angle.

10) $5\vec{i} - 12\vec{j}$



11) $\overrightarrow{RS}$ where $R = (-9, -1)$ $S = (-7, -3)$



$\|rs\| = 2\sqrt{2}$ or 2.83

Lesson 3.15 - Direction Angles

14

Practice — **In Exercises 61–66, find the magnitude and direction angle of the vector v.**

~~**61.** $\mathbf{v} = 5(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$~~

~~**62.** $\mathbf{v} = 8(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$~~

63. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

$\|v\| = 6\sqrt{2}$

$\theta = 315°$

64. $\mathbf{v} = -4\mathbf{i} - 7\mathbf{j}$

$\|v\| = \sqrt{65}$

$\theta = 240.3°$

65. $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$

$\|v\| = \sqrt{29}$

$\theta = 111.8°$

66. $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$

$\|v\| = \sqrt{369}$ or $3\sqrt{41}$

$\theta = 240.3°$

In Exercises 67–72, find the component form of v given its magnitude and the angle it makes with the positive x-axis. Sketch v.

	Magnitude	*Angle*
67.	$\|\mathbf{v}\| = 3$	$\theta = 0°$

$\langle 3, 0 \rangle$

68.	$\|\mathbf{v}\| = 1$	$\theta = 45°$

$\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$

69.	$\|\mathbf{v}\| = 3\sqrt{2}$	$\theta = 150°$

$\left\langle \frac{-3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right\rangle$

70.	$\|\mathbf{v}\| = 4\sqrt{3}$	$\theta = 90°$

$\langle 0, 4\sqrt{3} \rangle$

~~**71.** $\|\mathbf{v}\| = 2$ — $\mathbf{v}$ in the direction $\mathbf{i} + 3\mathbf{j}$~~

~~**72.** $\|\mathbf{v}\| = 3$ — $\mathbf{v}$ in the direction $3\mathbf{i} + 4\mathbf{j}$~~

3.14

Lesson 3.15 - Direction Angles

Practice **In Exercises 73–76, find the component form of the sum of u and v with direction angles θ_u and θ_v.**

	Magnitude	*Angle*
73.	$\|\mathbf{u}\| = 5$	$\theta_u = 60°$
	$\|\mathbf{v}\| = 5$	$\theta_v = 90°$

$u = \langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \rangle$

$v = \langle 0, 5 \rangle$

$u + v = \langle \frac{5}{2}, \frac{10+5\sqrt{3}}{2} \rangle$

	Magnitude	*Angle*
75.	$\|\mathbf{u}\| = 20$	$\theta_u = 45°$
	$\|\mathbf{v}\| = 50$	$\theta_v = 150°$

$u = \langle 10\sqrt{2}, 10\sqrt{2} \rangle$

$v = \langle -25\sqrt{3}, 25 \rangle$

$u + v = \langle 10\sqrt{2} - 25\sqrt{3}, 10\sqrt{2} + 25 \rangle$

74.	$\|\mathbf{u}\| = 2$	$\theta_u = 30°$
	$\|\mathbf{v}\| = 2$	$\theta_v = 90°$

$u = \langle \sqrt{3}, 1 \rangle$

$v = \langle 0, 2 \rangle$

$u + v = \langle \sqrt{3}, 3 \rangle$

76.	$\|\mathbf{u}\| = 35$	$\theta_u = 25°$
	$\|\mathbf{v}\| = 50$	$\theta_v = 120°$