Lesson 3.14 - Direction Angles

Learning Objectives: SWBAT

1. Determine the direction angle of a given vector
2. Perform operations on linear combinations of vectors

What is a direction Angle?

- A direction angle is an angle that is created by a vector
- Direction angles are always measured counterclockwise from the positive side of the x axis to the terminal point of the vector
- The components of the vector determine the "run" (x coordinate) and rise (y coordinate) of the vector. These coordinates are the legs of a right triangle and are "connected" by the direction angle
- If we use the direction angle as a reference angle of the right triangle, then in order to determine the measure of the angle, we can use the Tan^{-1} function

Examples - How to determine the direction angle of a vector

Find the direction angle of each vector.

a. $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j}$ **b.** $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$

Solution

a. The direction angle is

$$\tan\theta = \frac{b}{a} = \frac{3}{3} = 1. \qquad \tan^{-1}(1) = 45^\circ$$

So, $\theta = 45°$, as shown in Figure 6.29.

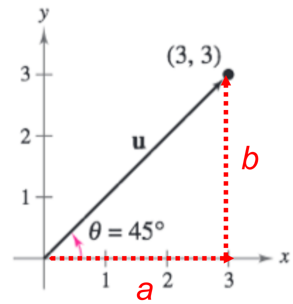

Figure 6.29

b. The direction angle is

$$\tan\theta = \frac{b}{a} = \frac{-4}{3}.$$

Moreover, because $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ lies in Quadrant IV, θ lies in Quadrant IV and its reference angle is

$$\theta' = \left|\tan^{-1}\left(-\frac{4}{3}\right)\right| \approx |-53.13°| = 53.13°.$$

So, it follows that $\theta \approx 360° - 53.13° = 306.87°$, as shown in Figure 6.30.

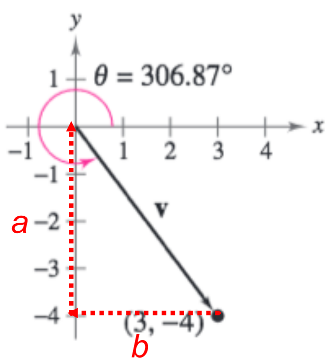

Figure 6.30

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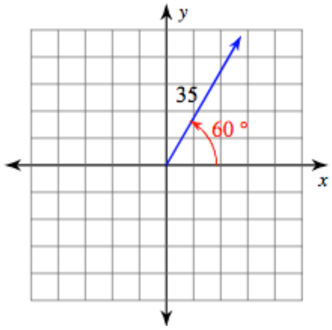
Practice

Write each vector in component form.

3)



4) $|\vec{k}| = 52,\ 174°$

Draw a diagram to illustrate the horizontal and vertical components of the vector. Then find the magnitude of each component.

5) $|\vec{t}| = 26,\ 115°$

6) $|\vec{a}| = 15,\ 230°$

Find the magnitude and direction angle for each vector.

7) $8\vec{i} + 15\vec{j}$

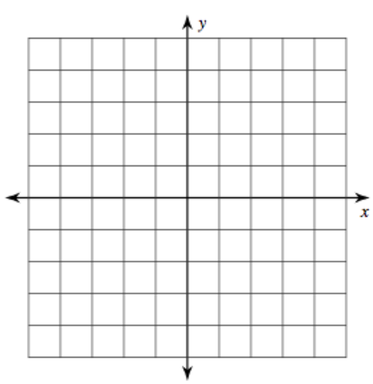
8) $\vec{r} = \langle -8, -41 \rangle$

Find the component form, magnitude, and direction angle for the given vector

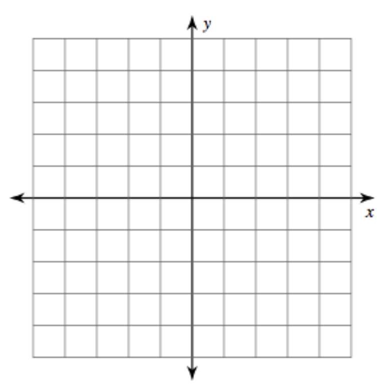
9) $\overrightarrow{CD}$ where $C = (6, -3)$ $D = (-6, -9)$

Sketch a graph of each vector then find the magnitude and direction angle.

10) $5\vec{i} - 12\vec{j}$



11) $\overrightarrow{RS}$ where $R = (-9, -1)$ $S = (-7, -3)$



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Practice **In Exercises 61–66, find the magnitude and direction angle of the vector v.**

61. $\mathbf{v} = 5(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

62. $\mathbf{v} = 8(\cos 135°\mathbf{i} + \sin 135°\mathbf{j})$

63. $\mathbf{v} = 6\mathbf{i} - 6\mathbf{j}$

64. $\mathbf{v} = -4\mathbf{i} - 7\mathbf{j}$

65. $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$

66. $\mathbf{v} = 12\mathbf{i} + 15\mathbf{j}$

In Exercises 67–72, find the component form of v given its magnitude and the angle it makes with the positive x-axis. Sketch v.

	Magnitude	*Angle*
67.	$\|\mathbf{v}\| = 3$	$\theta = 0°$
68.	$\|\mathbf{v}\| = 1$	$\theta = 45°$
69.	$\|\mathbf{v}\| = 3\sqrt{2}$	$\theta = 150°$
70.	$\|\mathbf{v}\| = 4\sqrt{3}$	$\theta = 90°$
71.	$\|\mathbf{v}\| = 2$	$\mathbf{v}$ in the direction $\mathbf{i} + 3\mathbf{j}$
72.	$\|\mathbf{v}\| = 3$	$\mathbf{v}$ in the direction $3\mathbf{i} + 4\mathbf{j}$

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Practice **In Exercises 73–76, find the component form of the sum of u and v with direction angles θ_u and θ_v.**

	Magnitude	*Angle*
73.	$\|\mathbf{u}\| = 5$	$\theta_u = 60°$
	$\|\mathbf{v}\| = 5$	$\theta_v = 90°$
74.	$\|\mathbf{u}\| = 2$	$\theta_u = 30°$
	$\|\mathbf{v}\| = 2$	$\theta_v = 90°$
75.	$\|\mathbf{u}\| = 20$	$\theta_u = 45°$
	$\|\mathbf{v}\| = 50$	$\theta_v = 150°$
76.	$\|\mathbf{u}\| = 35$	$\theta_u = 25°$
	$\|\mathbf{v}\| = 50$	$\theta_v = 120°$