Lesson 3.1 - Degrees, Minutes and Seconds

Learning Objectives: SWBAT

1. Define Trigonometry
2. Convert angles measures in decimal degrees notation (DD) to degrees, minutes, seconds notation (DMS)
3. Add/subtract two angle measures given in degrees, minutes, seconds notation

Review of basic Vocabulary:

- Trigonometry is the study of _____ _____
- Complementary angles add to _____ degrees
- Supplementary angles add to _____ degrees

Angle Measurements

An angle is formed by rotating a given ray about its endpoint to some terminal position. The original ray is the initial side of the angle and the second ray is the terminal side of the angle. The common endpoint is the vertex of the angle (see figure below). The measure of an angle is determined by the amount of rotation of the terminal ray from the initial ray.

For the purposes of this worksheet we will discuss two ways to measure angles: by degrees and by radians.

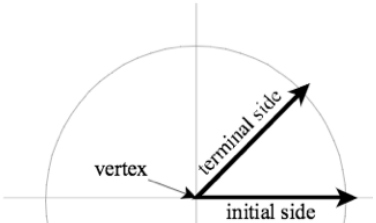

Figure 1

Degrees

The concept of measuring angles in *degrees* grew out of the belief of the early Babylonians that the seasons repeated every 360 days. One degree is the measure of an angle formed by rotating a ray (one three hundred sixtieth) of a complete revolution.

There are two popular methods for representing degrees and their fractional parts. One is the decimal degree method. For example, the measure 29.76° is a decimal degree. It means

29° plus 76 hundredths of 1°

A second method of measurement is known as the DMS (Degree, Minute, Second) method. For example, the measure 126° 12′ 27″ is a degree value expressed in DMS form. It means

126° plus 12 minutes plus 27 seconds

In the DMS method the fractional part of a degree may be expressed by understanding that we subdivide a degree into 60 equal parts, each of which is called a minute (denoted by ′) and that a minute is subdivided into 60 equal parts, each of which is called a second (denoted by ″). Thus $1° = 60′$, $1′ = 60″$, and $1° = 3600″$.

Changing Minutes and Seconds to Decimal Degrees: It is sometimes necessary to change minutes or seconds to decimal equivalents or vice versa. Minutes or seconds are first changed to their fractional part of a degree. Then the fraction is changed to its decimal equivalent by dividing the numerator by the denominator.

$$\text{Remember: } 1' = \frac{1}{60} \text{ of a degree, and } 1'' = \frac{1}{3600}$$

- *To change minutes to a decimal part of a degree:* **Divide minutes by 60.**
- *To change seconds to a decimal part of a degree:* **Divide seconds by 3600.**

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For example: Convert 50° 15′ 27″ to a decimal degree value.

Change 15′ to its decimal degree equivalent. $\frac{15}{60}$ $= 0.25°$

Change 27″ to its decimal degree equivalent. $\frac{27}{3600}$ $= 0.0075°$

And then add the values together: $50° + 0.25° + 0.0075°$ $= \mathbf{50.2575°}$

Your Turn #1: Convert the measure 37° 45' 17" to a decimal degree value

Changing a Decimal Degree into a DMS Degree Value: The decimal part of a degree can be changed to minutes and seconds by reversing the procedure. To change a decimal part of a degree to minutes, multiply by 60. Similarly, to change the decimal part of a minute to seconds, multiply by 60.

- *To change a decimal part of a degree to minutes:* **Multiply the decimal part of a degree by 60.**
- *To change a decimal part of a minute to seconds:* **Multiply the decimal part of a minute by 60.**

For example: Convert 50.75° into a DMS degree value

Change 0.75° to minutes	0.75×60	$= 45'$
And so...	50.75°	$= \mathbf{50° 45'}$

For example: Convert 28.43° into a DMS degree value

Change 0.43° to minutes and seconds	0.43×60	$= 25.8'$ (*degrees to minutes*)
	0.8×60	$= 48''$ (*decimal part of min to sec*)
And so...	28.43°	$= \mathbf{28° 25' 48''}$

Your Turn #2: Convert the measure 36.69° to minutes, degrees seconds

Adding and Subtracting Angle Measures: Angle measures can be added or subtracted. Keep in mind that only like measures can be added or subtracted.

To add, arrange the measures in columns of like measures.

For example:

$$\begin{array}{rrr} 12° & 15' & 54'' \\ +\ 82° & 28' & 19'' \\ \hline 94° & 43' & 73'' \end{array}$$

which simplifies* to **94° 44' 13″**

*Since 73″ = 1' 13″, we must simplify and shift the values to the left as appropriate. In this case we must add one minute to 43 to make 44 minutes leaving a remainder of 13 seconds.

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Your Turn #3: Add the following Angle measures: $(34^\circ\ 47'\ 32'') + (18^\circ\ 37'\ 51'')$

To subtract, arrange the measures in columns of like measures; borrow as needed.

For example:

$$\begin{array}{r} 37^\circ\ 15' = 36^\circ\ 75' \\ -15^\circ\ 32' = -15^\circ\ 32' \\ \hline 21^\circ\ 43' \end{array}$$

(Borrow 1° from 37°. 1° = 60′ and 60′ + 15′ = 75′.)

Since we can't subtract 32 from 15 with real numbers, we must borrow (much like you would in regular whole number subtraction) one degree to make more minutes from which to subtract.

Your Turn #4: Subtract the following Angle measures: $(96^\circ\ 31'\ 12'') - (42^\circ\ 54'\ 45'')$

Practice:

1. Change 0.42° to equivalent minutes and seconds.

2. Change 15° 4' to its decimal degree equivalent rounded to the nearest ten-thousandth.

3. Change 0.46° to equivalent minutes and seconds.

4. Change 8° 20' to its decimal degree equivalent rounded to the nearest ten-thousandth.

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Practice:

5) Add and simplify:
$$\begin{array}{r} 15^\circ\,47'\,18'' \\ +\,37^\circ\,12'\,45'' \\ \hline \end{array}$$

6) Subtract and simplify:
$$\begin{array}{r} 147^\circ\,28' \\ -\,114^\circ\,35'\,23'' \\ \hline \end{array}$$

7) Add and simplify:
$$\begin{array}{r} 45^\circ\,10'\,14'' \\ +\;\;7^\circ\;\;8'\,55'' \\ \hline \end{array}$$

8) Subtract and simplify:
$$\begin{array}{r} 32^\circ \quad\; 6'' \\ -\,20^\circ\;\;10'\;\;8'' \\ \hline \end{array}$$

9. Find the measure of an angle with a complement of 35°.

10. Find the measure of an angle with a supplement of 35°.

11. An angle whose measure is 17° 36' 40" needs to be three times as large. Find the measure of the new angle in degrees and minutes.

12. An angle whose measure is 45° 37' 30" needs to be twice as large. Find the measure of. the new angle in degrees and minutes.

13. A right angle will be divided into four equal angles. Find the measure of each new angle in degrees and minutes.

14. Find the complement of 40° 37' 26". Then convert the result to its decimal equivalent rounded to the nearest ten-thousandth.