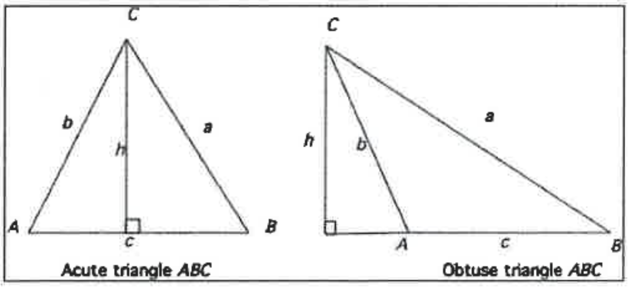
Lesson 3.8 - Area of an Oblique Triangle

Learning Objectives: SWBAT

1. Determine the area of an oblique triangle
2. Use Heron's formula to determine the area of an oblique triangle given SSS information

There are two formulas using trigonometry that will allow us to find the area of oblique triangles based on given information. Obviously, if the triangle is a right triangle, we only need both legs: Area $= \frac{1}{2}(\text{base})(\text{height})$.



Area $= \frac{1}{2}(\text{base})(\text{height})$

Area $= \frac{1}{2}cb\sin A$

So we say Area $= \frac{1}{2}bc\sin A$

Similarly, Area $= \frac{1}{2}ac\sin B$

Similarly, Area $= \frac{1}{2}ab\sin C$

This formula works when you have two sides and the included angle (SAS). But frequently you have three sides of a triangle and wish to determine the area. In that case, we have another formula that will determine the area of that triangle. It is called Heron's (pronounced Hero's) formula.

Heron's Formula

If ΔABC has sides $a, b,$ and c, the area of the triangle is given by

Area $= \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

Example 1 - Determine the area of the oblique triangle given the following information: **∠A = 38°, ∠B = 57°, c = 16 cm**

- Use 180° rule to determine that **∠C = 85°**
- Use law of sines to determine a or b $\frac{a}{\sin 38} = \frac{16}{\sin 85}$ **a = 9.89 cm**
- Plug information into area formula **Area** $= \frac{1}{2}ac\sin B$

> $\frac{1}{2}(9.89)(16)(\sin 57) =$ Area = 66.36 cm²

Your Turn: Determine the area of the oblique triangle given the following information: **∠A = 70°, ∠C = 30°, c = 32 cm**

∡B = 80°

b

$\frac{32}{\sin 30} = \frac{b}{\sin 80}$

b = 63.03

→ $\frac{1}{2}cb\sin A$

given, need to find first, given

Area $= \frac{1}{2}(32)(63.03)(\sin 70)$

↓

947.66 cm²

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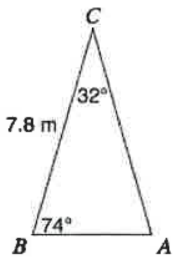
Example 2 - Determine the area of the oblique triangle given the following information: **a = 12 cm, b = 15 cm, c = 25 cm**

- Determine $\frac{a+b+c}{2}$ $\quad \frac{12+15+25}{2} = 26 = s$
- Plug into Heron's Formula $\sqrt{26(26-12)(26-15)(26-25)}$ **Area = 66.36 cm²**

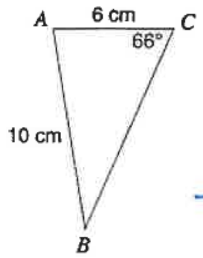
Practice: Determine the area of the following oblique triangles (round answer to the nearest tenth, please don't forget labels

1)



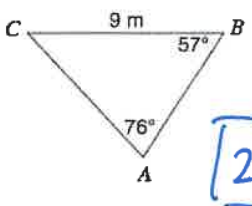
16.1 m²

2)



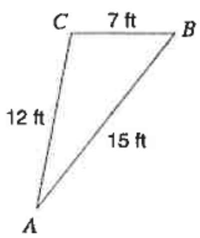
29.6 cm²

3)



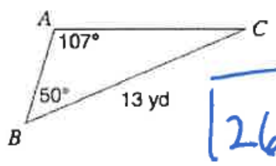
25.7 m2

4)



41.2 ft²

5)



26.4 yd²

6) $b = 13$ cm, $c = 11$ cm, $a = 5$ cm

26.9 cm²

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7) $m\angle B = 137°$, $a = 5.9$ mi, $m\angle C = 28°$

21.5 mi^2

8) $m\angle C = 137°$, $m\angle A = 24°$, $b = 4$ cm

6.8 cm^2

9) $m\angle C = 62°$, $b = 7$ yd, $a = 10$ yd

30.9 yd^2

10) $a = 9$ mi, $m\angle B = 27°$, $m\angle A = 137°$

7.4 mi^2

11) $a = 4.3$ in, $c = 13$ in, $b = 14$ in

27.9 in^2

12) $m\angle C = 101°$, $c = 4$ mi, $m\angle A = 56°$

2.7 mi^2

13) $c = 9.2$ ft, $m\angle A = 106°$, $a = 15$ ft

42.4 ft^2

14) $m\angle C = 60°$, $b = 6.9$ in, $a = 4$ in

12 in^2

15) $c = 12.1$ km, $m\angle C = 107°$, $b = 6.6$ km

26.5 km^2