Lesson 4.1 - Radian Measure

Learning Objectives: SWBAT

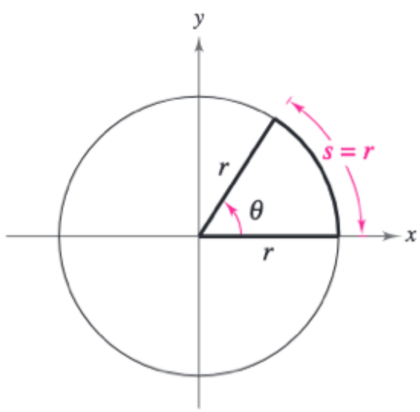
1. Explain the difference between degree and radian measure of angles
2. Draw rough sketch of reference angles given in degrees or radians
3. Convert angle measures from degrees to radians and vice versa

Making a connection - Overview of Unit 4: Circular Functions

- The Trigonometric functions (Sine, Cosine, Tangent) that we studied in unit 3 were based on the relationship between the size of a given reference angle and ratios of the sides of a triangle that can be made with that angle
- These trigonometric functions are also called circular functions because he reference angles can also be related to the arc that they create
- This unit will further explore the relationship between reference angles and the circles/triangles that they create

What are Radians?

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*. This type of measure is especially useful in calculus. To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle, as shown in Figure 4.5.



Arc length = radius when $\theta = 1$ radian.
Figure 4.5

Definition of Radian

One **radian** (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians.

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What is the basis for radian measure?

Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of

$$s = 2\pi r.$$

Moreover, because $2\pi \approx 6.28$, there are just over six radius lengths in a full circle, as shown in Figure 4.6. Because the units of measure for s and r are the same, the ratio s/r has no units—it is simply a real number.

Because the radian measure of an angle of one full revolution is 2π, you can obtain the following.

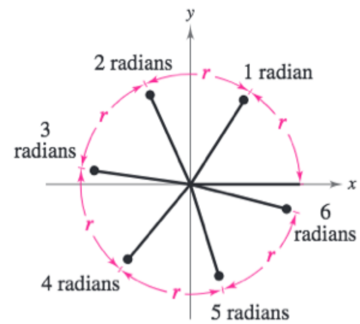


Figure 4.6

$$\frac{1}{2}\text{ revolution} = \frac{2\pi}{2} = \pi \text{ radians}$$

$$\frac{1}{4}\text{ revolution} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ radians}$$

$$\frac{1}{6}\text{ revolution} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}$$

These and other common angles are shown in Figure 4.7.

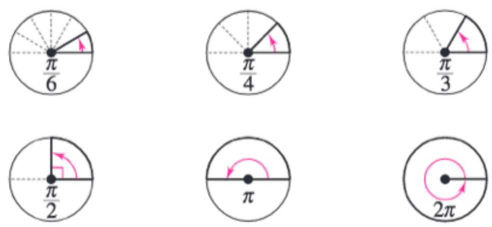


Figure 4.7

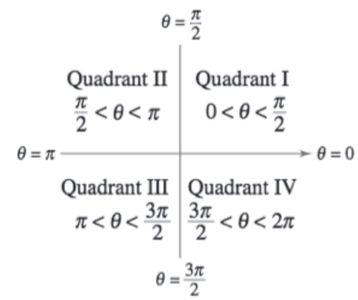
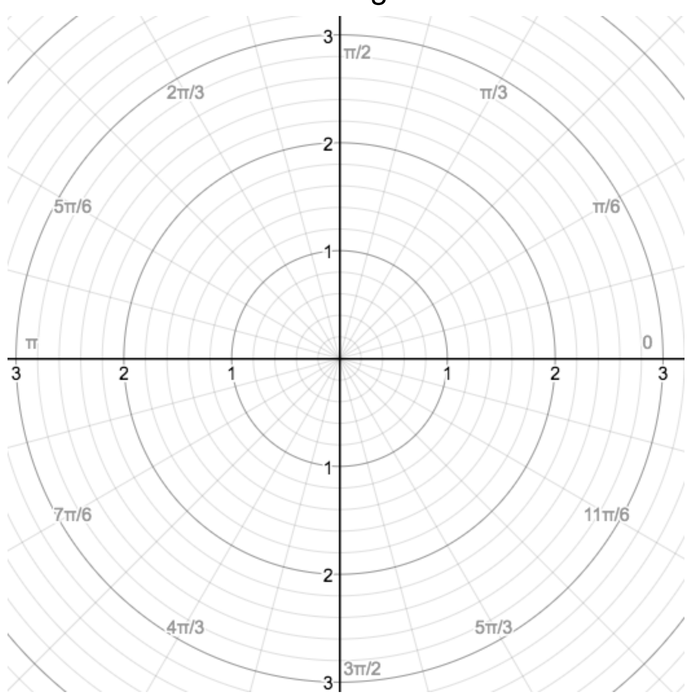


Figure 4.8

Recall that the four quadrants in a coordinate system are numbered I, II, III, and IV. Figure 4.8 shows which angles between 0 and 2π lie in each of the four quadrants. Note that angles between 0 and $\pi/2$ are **acute** and that angles between $\pi/2$ and π are **obtuse.**

The following provides you of a coordinate plane that demonstrates the radian measures for the common reference angles



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Converting degrees to Radians and vice versa

Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi \text{ rad}}{180°}$.

2. To convert radians to degrees, multiply radians by $\dfrac{180°}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship π rad $= 180°$. (See Figure 4.14.)

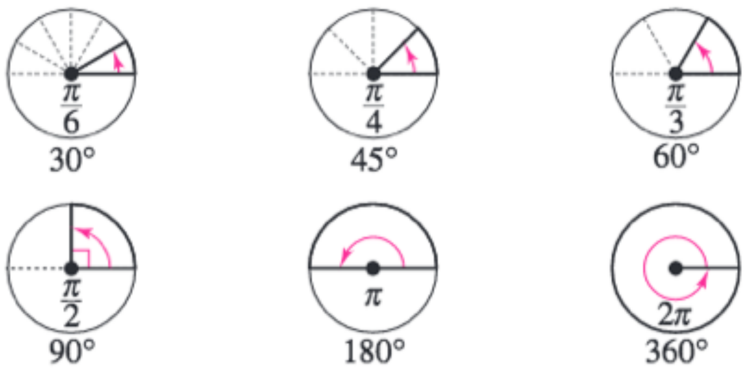

Figure 4.14

When no units of angle measure are specified, *radian measure is implied*. For instance, if you write $\theta = \pi$ or $\theta = 2$, you imply that $\theta = \pi$ radians or $\theta = 2$ radians.

Example 3 Converting from Degrees to Radians

a. $135° = (135 \cancel{\text{deg}})\left(\dfrac{\pi \text{ rad}}{180 \cancel{\text{deg}}}\right) = \dfrac{3\pi}{4}$ radians Multiply by $\frac{\pi}{180}$.

b. $540° = (540 \cancel{\text{deg}})\left(\dfrac{\pi \text{ rad}}{180 \cancel{\text{deg}}}\right) = 3\pi$ radians Multiply by $\frac{\pi}{180}$.

c. $-270° = (-270 \cancel{\text{deg}})\left(\dfrac{\pi \text{ rad}}{180 \cancel{\text{deg}}}\right) = -\dfrac{3\pi}{2}$ radians Multiply by $\frac{\pi}{180}$.

Example 4 Converting from Radians to Degrees

a. $-\dfrac{\pi}{2} \text{ rad} = \left(-\dfrac{\pi}{2} \cancel{\text{rad}}\right)\left(\dfrac{180 \text{ deg}}{\pi \cancel{\text{rad}}}\right) = -90°$ Multiply by $\frac{180}{\pi}$.

b. $2 \text{ rad} = (2 \cancel{\text{rad}})\left(\dfrac{180 \text{ deg}}{\pi \cancel{\text{rad}}}\right) = \dfrac{360}{\pi} \approx 114.59°$ Multiply by $\frac{180}{\pi}$.

c. $\dfrac{9\pi}{2} \text{ rad} = \left(\dfrac{9\pi}{2} \cancel{\text{rad}}\right)\left(\dfrac{180 \text{ deg}}{\pi \cancel{\text{rad}}}\right) = 810°$ Multiply by $\frac{180}{\pi}$.

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Practice

In Exercises 3–6, determine the quadrant in which each angle lies. (The angle measure is given in radians.)

3. (a) $\frac{7\pi}{4}$ (b) $\frac{11\pi}{4}$

4. (a) $-\frac{5\pi}{12}$ (b) $-\frac{13\pi}{9}$

5. (a) -1 (b) -2

6. (a) 3.5 (b) 2.25

In Exercises 7–10, sketch each angle in standard position.

7. (a) $\frac{3\pi}{4}$ (b) $\frac{4\pi}{3}$

8. (a) $-\frac{7\pi}{4}$ (b) $-\frac{5\pi}{2}$

9. (a) $\frac{11\pi}{6}$ (b) $\frac{2\pi}{3}$

10. (a) 4 (b) -3

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In Exercises 23–26, determine the quadrant in which each angle lies.

23. (a) $150°$ (b) $282°$

24. (a) $87.9°$ (b) $8.5°$

25. (a) $-132° 50'$ (b) $-336° 30'$

26. (a) $-245.25°$ (b) $-12.35°$

In Exercises 39–42, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

39. (a) $30°$ (b) $150°$

40. (a) $315°$ (b) $120°$

41. (a) $-20°$ (b) $-240°$

42. (a) $-270°$ (b) $144°$

In Exercises 43–46, rewrite each angle in degree measure. (Do not use a calculator.)

43. (a) $\frac{3\pi}{2}$ (b) $-\frac{7\pi}{6}$

44. (a) -4π (b) 3π

45. (a) $\frac{7\pi}{3}$ (b) $-\frac{13\pi}{60}$

46. (a) $-\frac{15\pi}{6}$ (b) $\frac{28\pi}{15}$

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In Exercises 47–52, convert the angle measure from degrees to radians. Round your answer to three decimal places.

47. $115°$

48. $83.7°$

49. $-216.35°$

50. $-46.52°$

51. $-0.78°$

52. $395°$

In Exercises 53–58, convert the angle measure from radians to degrees. Round your answer to three decimal places.

53. $\frac{\pi}{7}$

54. $\frac{8\pi}{13}$

55. 6.5π

56. -4.2π

57. -2

58. -0.48