Lesson 4.2 - Coterminal angles

Learning Objectives: SWBAT

1. Determine and sketch angles that are coterminal to a given angle
2. Determine complimentary and supplementary angles (in radians)

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1. The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive x-axis. Such an angle is in **standard position,** as shown in Figure 4.2. **Positive angles** are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.3. Angles are labeled with Greek letters such as α (alpha), β (beta), and θ (theta), as well as uppercase letters such as A, B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**

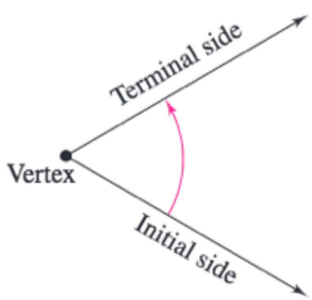


Figure 4.1

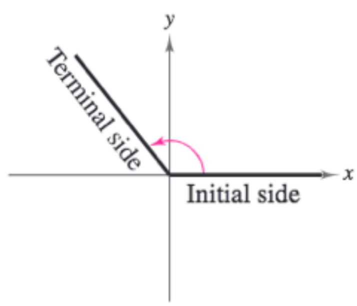


Figure 4.2

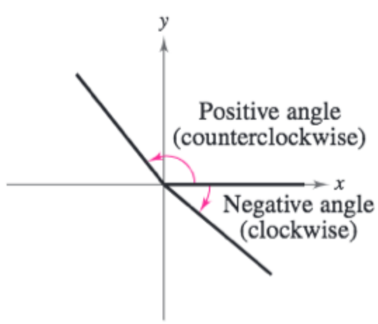


Figure 4.3

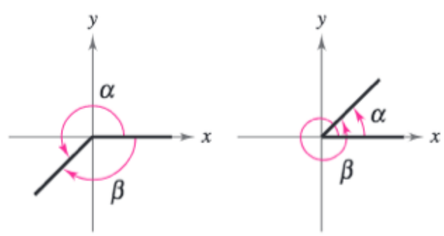


Figure 4.4

Two positive angles α and β are **complementary** (complements of each other) if their sum is $\pi/2$. Two positive angles are **supplementary** (supplements of each other) if their sum is π. See Figure 4.12.

Complementary angles *Supplementary angles*

Figure 4.12

Lesson 4.2 - Coterminal angles

Examples: How to find coterminal angle(s) of a given angle

a. For the positive angle $\theta = \frac{13\pi}{6}$, subtract 2π to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$

See Figure 4.9.

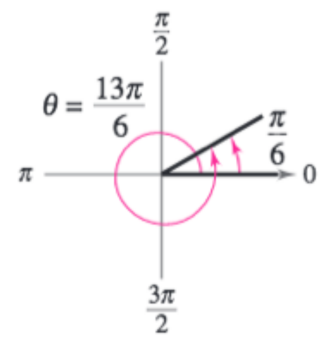


Figure 4.9

b. For the positive angle $\theta = \frac{3\pi}{4}$, subtract 2π to obtain a coterminal angle

$$\frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}.$$

See Figure 4.10.

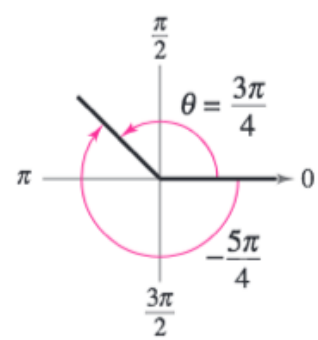


Figure 4.10

c. For the negative angle $\theta = -\frac{2\pi}{3}$, add 2π to obtain a coterminal angle

$$-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}.$$

See Figure 4.11.

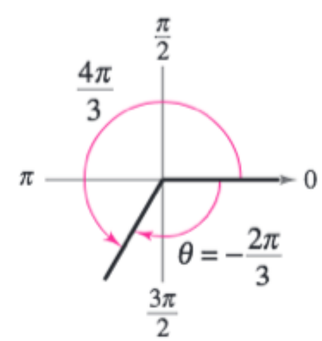


Figure 4.11

Your Turn: For the negative angle $-\frac{5\pi}{4}$, determine two coterminal angles

Lesson 4.2 - Coterminal angles

Examples: How to find complimentary and supplementary angles (in radians)

If possible, find the complement and the supplement of (a) $\frac{2\pi}{5}$ and (b) $\frac{4\pi}{5}$.

Solution

a. The complement of $\frac{2\pi}{5}$ is

$$\frac{\pi}{2} - \frac{2\pi}{5} = \frac{5\pi}{10} - \frac{4\pi}{10}$$

$$= \frac{\pi}{10}.$$

The supplement of $\frac{2\pi}{5}$ is

$$\pi - \frac{2\pi}{5} = \frac{5\pi}{5} - \frac{2\pi}{5}$$

$$= \frac{3\pi}{5}.$$

b. Because $4\pi/5$ is greater than $\pi/2$, it has no complement. (Remember that complements are *positive* angles.) The supplement is

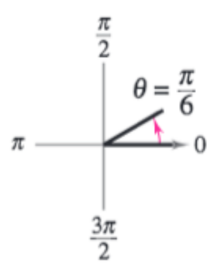
$$\pi - \frac{4\pi}{5} = \frac{5\pi}{5} - \frac{4\pi}{5}$$

$$= \frac{\pi}{5}.$$

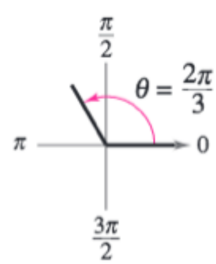
Practice:

In Exercises 11–14, determine two coterminal angles in radian measure (one positive and one negative) for each angle. (There are many correct answers).

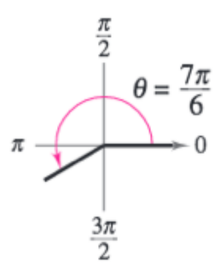
11. (a)



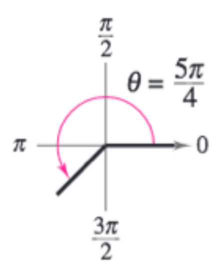
(b)



12. (a)



(b)



13. (a) $-\frac{9\pi}{4}$ (b) $-\frac{2\pi}{15}$

14. (a) $\frac{7\pi}{8}$ (b) $\frac{8\pi}{45}$

Lesson 4.2 - Coterminal angles

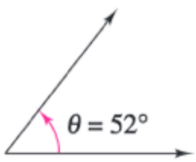
Practice:

In Exercises 15–20, find (if possible) the complement and supplement of the angle.

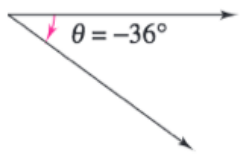
15. $\frac{\pi}{3}$ **16.** $\frac{3\pi}{4}$ **17.** $\frac{\pi}{6}$ **18.** $\frac{2\pi}{3}$

In Exercises 31–34, determine two coterminal angles in degree measure (one positive and one negative) for each angle. (There are many correct answers).

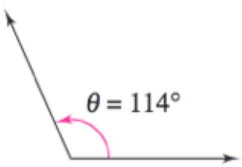
31. (a)



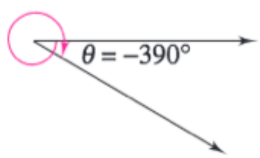
(b)



32. (a)



(b)



33. (a) $300°$ (b) $230°$

34. (a) $-445°$ (b) $-740°$

75. Find each angle (in radians) shown on the unit circle.