Lesson 4.3 - Prelude

Library of Parent Functions: Trigonometric Functions

Trigonometric functions are transcendental functions. The six trigonometric functions, sine, cosine, tangent, cosecant, secant, and cotangent, have important uses in construction, surveying, and navigation. Their periodic behavior makes them useful for modeling phenomena such as business cycles, planetary orbits, pendulums, wave motion, and light rays.

The six trigonometric functions can be defined in three different ways.

1. As the ratio of two sides of a right triangle [see Figure 4.41(a)].
2. As coordinates of a point (x, y) in the plane and its distance r from the origin [see Figure 4.41(b)].
3. As functions of any real number, such as time t.

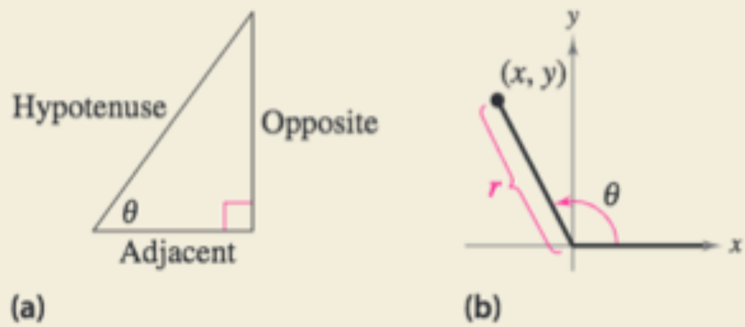
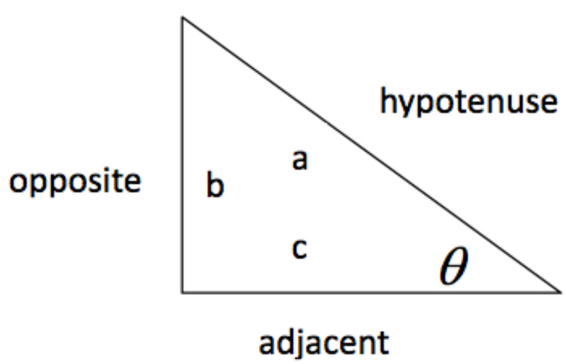


Figure 4.41

The Trigonometric Ratios



$$\sin\theta = \frac{opp.}{hyp.} = \frac{b}{a} \qquad \cos\theta = \frac{adj.}{hyp.} = \frac{c}{a}$$

$$\tan\theta = \frac{opp.}{adj.} = \frac{b}{c} \qquad \cot\theta = \frac{adj.}{opp.} = \frac{c}{b}$$

$$\sec\theta = \frac{hyp.}{adj.} = \frac{a}{c} \qquad \csc\theta = \frac{hyp.}{opp.} = \frac{a}{b}$$

Lesson 4.3 - Evaluating Trig Functions of any angle

Learning Objectives: SWBAT

1. Evaluate trigonometric functions of any angle

<u>Making a connection</u>

- In unit 3 we used sine, cosine, tangent to determine ratios of sides that corresponded with acute angles of a right triangle.
- In this lesson, we will relate those concepts to evaluate (determine ratios) related to any angle

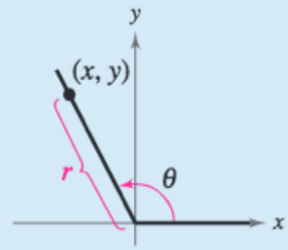
Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$\sin\theta = \frac{y}{r}$ $\qquad$ $\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x},\quad x \neq 0$ $\qquad$ $\cot\theta = \frac{x}{y},\quad y \neq 0$

$\sec\theta = \frac{r}{x},\quad x \neq 0$ $\qquad$ $\csc\theta = \frac{r}{y},\quad y \neq 0$



Because $r = \sqrt{x^2 + y^2}$ *cannot* be zero, it follows that the sine and cosine functions are defined for any real value of θ. However, if $x = 0$, the tangent and secant of θ are undefined. For example, the tangent of 90° is undefined. Similarly, if $y = 0$, the cotangent and cosecant of θ are undefined.

Example 1 Evaluating Trigonometric Functions

Let $(-3, 4)$ be a point on the terminal side of θ. Find the sine, cosine, and tangent of θ.

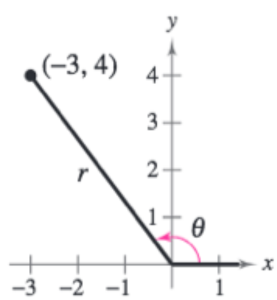


Figure 4.32

Solution

Referring to Figure 4.32, you can see that $x = -3$, $y = 4$, and

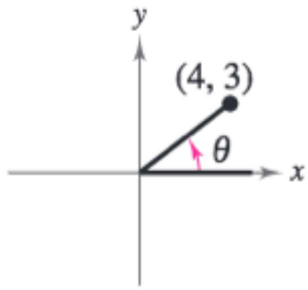
$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{25} = 5.$$

So, you have $\sin\theta = \frac{y}{r} = \frac{4}{5}$, $\cos\theta = \frac{x}{r} = -\frac{3}{5}$, and $\tan\theta = \frac{y}{x} = -\frac{4}{3}$.

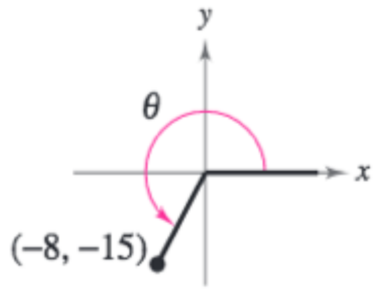
Lesson 4.3 - Evaluating Trig Functions of any angle

Practice **Library of Parent Functions** **In Exercises 1–4, determine the exact values of the six trigonometric functions of the angle θ.**

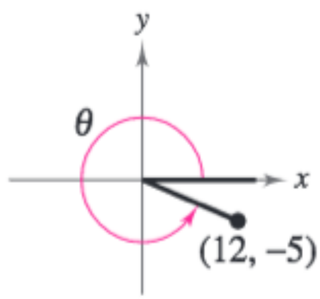
1. (a)



(b)



2. (a)



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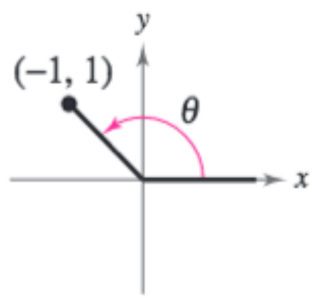
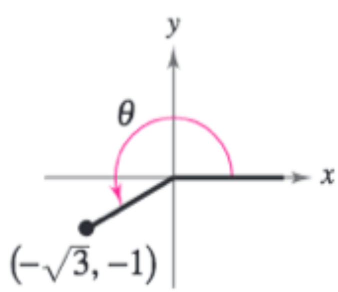
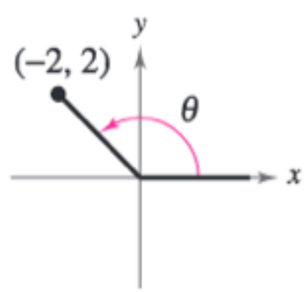


3. (a)



(b)



Lesson 4.3 - Evaluating Trig Functions of any angle

The *signs* of the trigonometric functions in the four quadrants can be determined easily from the definitions of the functions. For instance, because $\cos\theta = x/r$, it follows that $\cos\theta$ is positive wherever $x > 0$, which is in Quadrants I and IV. (Remember, r is always positive.) In a similar manner, you can verify the results shown in Figure 4.33.

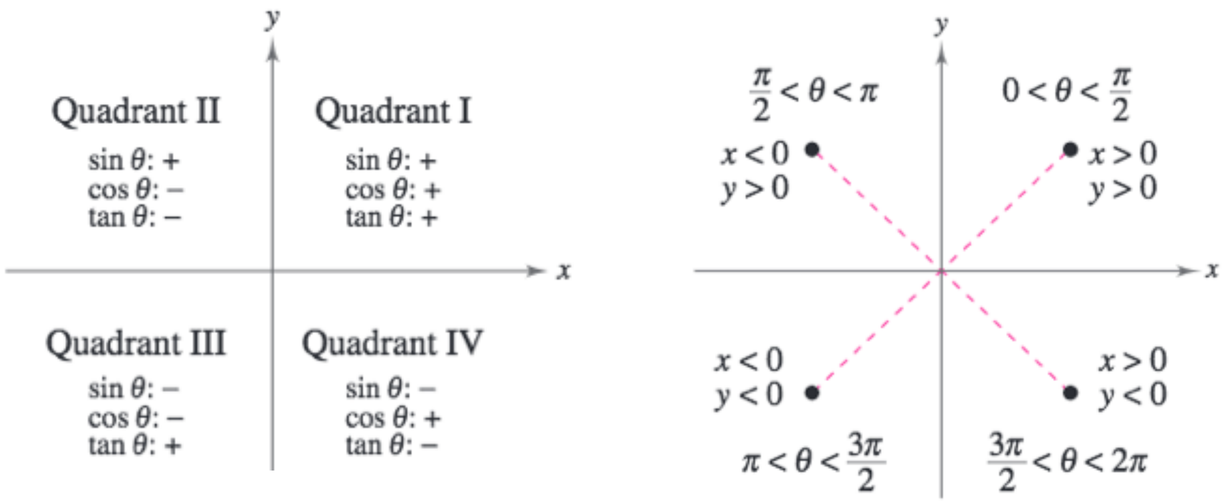


Figure 4.33

Example 2 Evaluating Trigonometric Functions

Given $\sin\theta = -\frac{2}{3}$ and $\tan\theta > 0$, find $\cos\theta$ and $\cot\theta$.

Solution

Note that θ lies in Quadrant III because that is the only quadrant in which the sine is negative and the tangent is positive. Moreover, using

$$\sin\theta = \frac{y}{r} = -\frac{2}{3}$$

and the fact that y is negative in Quadrant III, you can let $y = -2$ and $r = 3$. Because x is negative in Quadrant III, $x = -\sqrt{9-4} = -\sqrt{5}$, and you have the following.

$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{5}}{3}$$ Exact value

$$\approx -0.75$$ Approximate value

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{5}}{-2}$$ Exact value

$$\approx 1.12$$ Approximate value

Lesson 4.3 - Evaluating Trig Functions of any angle

Practice **In Exercises 13–16, state the quadrant in which θ lies.**

13. $\sin\theta < 0$ and $\cos\theta < 0$

14. $\sec\theta > 0$ and $\cot\theta < 0$

15. $\cot\theta > 0$ and $\cos\theta > 0$

16. $\tan\theta > 0$ and $\csc\theta < 0$

In Exercises 17–24, find the values of the six trigonometric functions of θ.

Function Value	*Constraint*
17. $\sin\theta = \frac{3}{5}$	θ lies in Quadrant II.
18. $\cos\theta = -\frac{4}{5}$	θ lies in Quadrant III.
19. $\tan\theta = -\frac{15}{8}$	$\sin\theta < 0$
20. $\csc\theta = 4$	$\cot\theta < 0$
22. $\sin\theta = 0$	$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

Lesson 4.3 - Evaluating Trig Functions of any angle

Example 3 Trigonometric Functions of Quadrant Angles

Evaluate the sine and cosine functions at the angles 0, $\frac{\pi}{2}$, π, and $\frac{3\pi}{2}$.

Solution

To begin, choose a point on the terminal side of each angle, as shown in Figure 4.34. For each of the four given points, $r = 1$, and you have the following.

$\sin 0 = \frac{y}{r} = \frac{0}{1} = 0$ $\cos 0 = \frac{x}{r} = \frac{1}{1} = 1$ $(x, y) = (1, 0)$

$\sin \frac{\pi}{2} = \frac{y}{r} = \frac{1}{1} = 1$ $\cos \frac{\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$ $(x, y) = (0, 1)$

$\sin \pi = \frac{y}{r} = \frac{0}{1} = 0$ $\cos \pi = \frac{x}{r} = \frac{-1}{1} = -1$ $(x, y) = (-1, 0)$

$\sin \frac{3\pi}{2} = \frac{y}{r} = \frac{-1}{1} = -1$ $\cos \frac{3\pi}{2} = \frac{x}{r} = \frac{0}{1} = 0$ $(x, y) = (0, -1)$

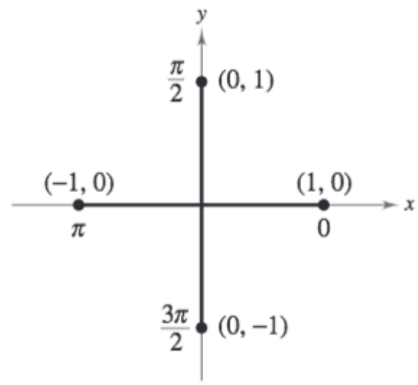


Figure 4.34

Practice

In Exercises 29–36, evaluate the trigonometric function of the quadrant angle.

29. $\sec \pi$

30. $\tan \frac{\pi}{2}$

31. $\cot \frac{3\pi}{2}$

32. $\csc 0$

33. $\sec 0$

34. $\csc \frac{3\pi}{2}$

35. $\cot \pi$

36. $\csc \frac{\pi}{2}$