

## Lesson 4.4 - Reference Angles

Learning Objectives: SWBAT

1. Determine/sketch the reference angle of a given angle

What are Reference Angles?

The values of the trigonometric functions of angles greater than  $90^\circ$  (or less than  $0^\circ$ ) can be determined from their values at corresponding acute angles called **reference angles**.

### Definition of Reference Angle

Let  $\theta$  be an angle in standard position. Its **reference angle** is the acute angle  $\theta'$  formed by the terminal side of  $\theta$  and the horizontal axis.

Figure 4.35 shows the reference angles for  $\theta$  in Quadrants II, III, and IV.

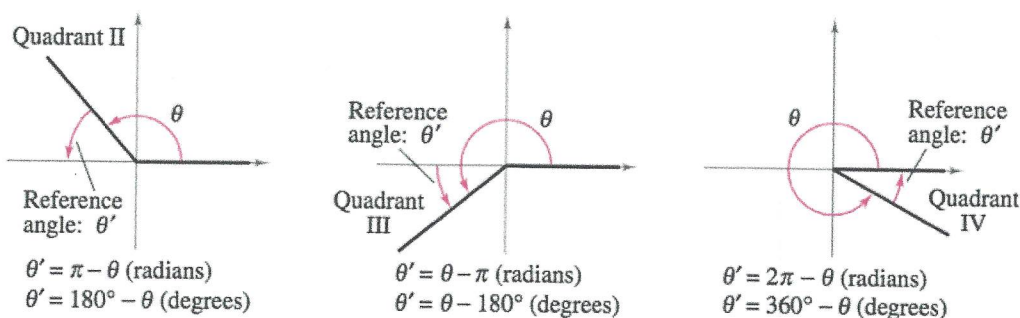


Figure 4.35

### How to find reference angles

Find the reference angle  $\theta'$ .

- a.  $\theta = 300^\circ$     b.  $\theta = 2.3$     c.  $\theta = -135^\circ$

### Solution

- a. Because  $300^\circ$  lies in Quadrant IV, the angle it makes with the  $x$ -axis is

$$\theta' = 360^\circ - 300^\circ = 60^\circ. \quad \text{Degrees}$$

- b. Because  $2.3$  lies between  $\pi/2 \approx 1.5708$  and  $\pi \approx 3.1416$ , it follows that it is in Quadrant II and its reference angle is

$$\theta' = \pi - 2.3 \approx 0.8416. \quad \text{Radians}$$

- c. First, determine that  $-135^\circ$  is coterminal with  $225^\circ$ , which lies in Quadrant III. So, the reference angle is

$$\theta' = 225^\circ - 180^\circ = 45^\circ. \quad \text{Degrees}$$

Figure 4.36 shows each angle  $\theta$  and its reference angle  $\theta'$ .

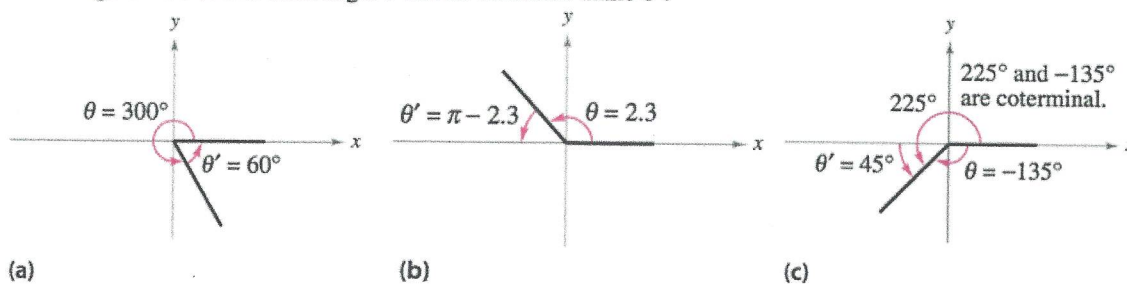


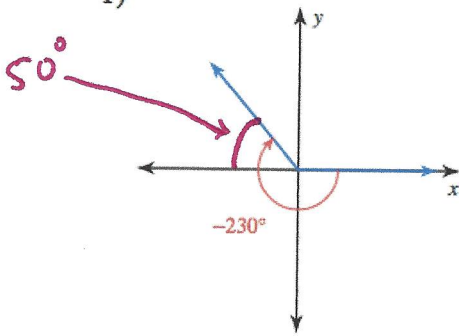
Figure 4.36

## Lesson 4.4 - Reference Angles

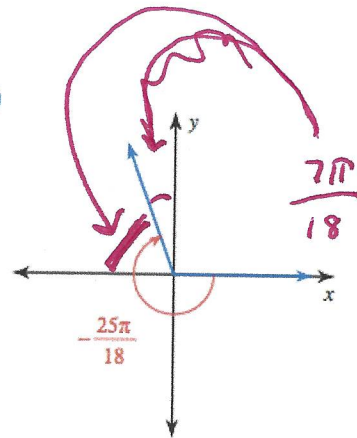
Practice - Find the reference angle for the following:

Find the reference angle.

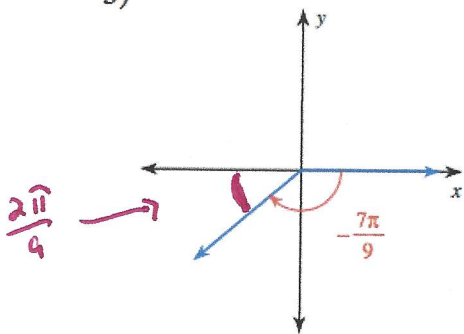
1)



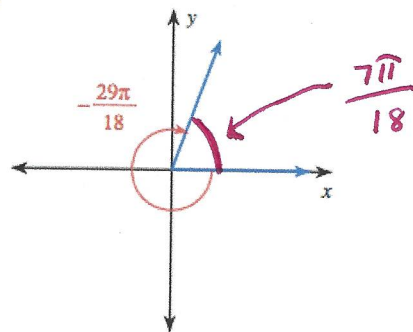
2)



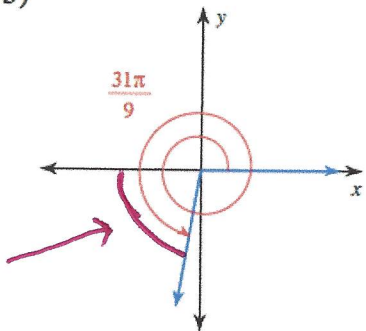
3)



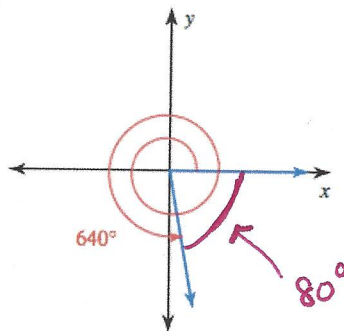
4)



5)

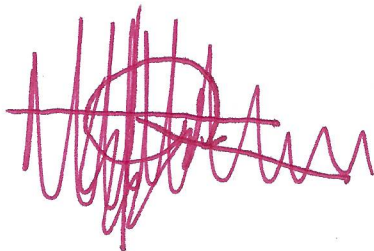


6)



$$\frac{\pi}{18}$$

8)  $-\frac{19\pi}{18}$



9)  $-\frac{13\pi}{12}$

$$\frac{\pi}{12}$$

$$\frac{\pi}{12}$$

9)  $-\frac{47\pi}{12}$

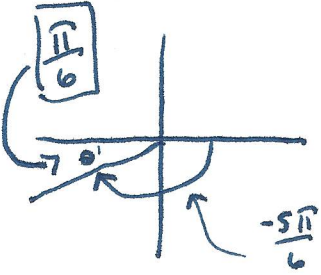
10)  $-\frac{5\pi}{4}$

$$\frac{\pi}{4}$$

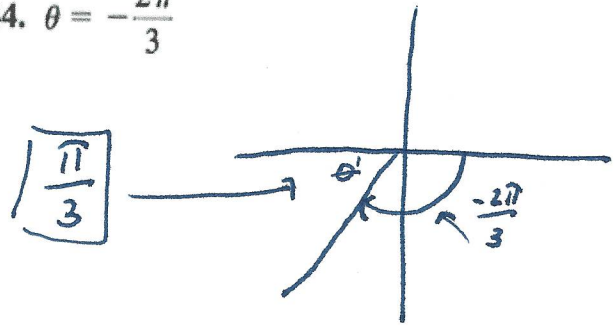
## Lesson 4.4 - Reference Angles

Practice - Sketch and find the reference angle for the following:

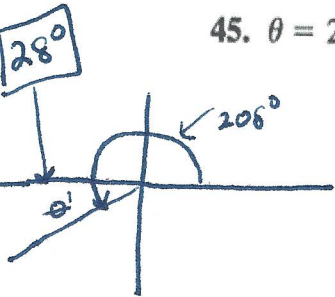
43.  $\theta = -\frac{5\pi}{6}$



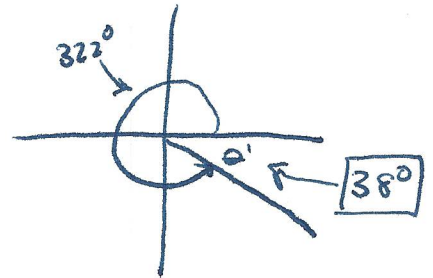
44.  $\theta = -\frac{2\pi}{3}$



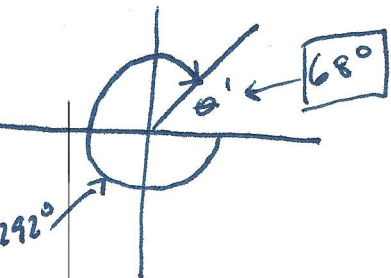
45.  $\theta = 208^\circ$



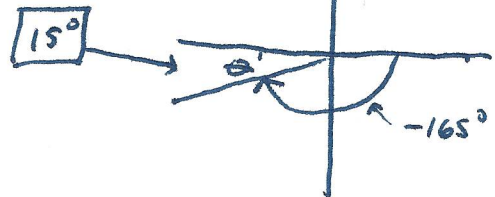
46.  $\theta = 322^\circ$



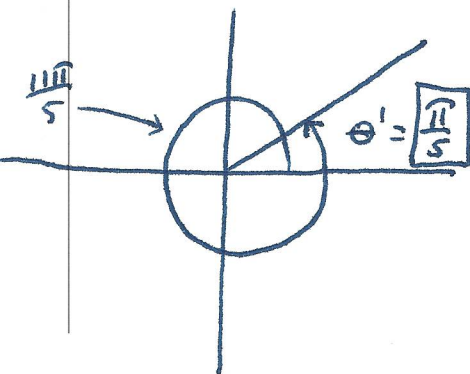
47.  $\theta = -292^\circ$



48.  $\theta = -165^\circ$



49.  $\theta = \frac{11\pi}{5}$



50.  $\theta = \frac{17\pi}{7}$

