

## Lesson 4.5 - Evaluating trig functions (part 2)

Learning Objectives: SWBAT

1. Use reference angles to evaluate trig functions for a given angle

### Making a connection

- In lesson 4.3, we learned to evaluate trig functions for a given angle if we were also given specific information about the quadrant it was in.
- In this lesson, we will use reference angles to determine the quadrant the angle is in before evaluating.

To see how a reference angle is used to evaluate a trigonometric function, consider the point  $(x, y)$  on the terminal side of  $\theta$ , as shown in Figure 4.37. By definition, you know that

$$\sin \theta = \frac{y}{r} \quad \text{and} \quad \tan \theta = \frac{y}{x}$$

For the right triangle with acute angle  $\theta'$  and sides of lengths  $|x|$  and  $|y|$ , you have

$$\sin \theta' = \frac{\text{opp}}{\text{hyp}} = \frac{|y|}{r}$$

and

$$\tan \theta' = \frac{\text{opp}}{\text{adj}} = \frac{|y|}{|x|}$$

So, it follows that  $\sin \theta$  and  $\sin \theta'$  are equal, *except possibly in sign*. The same is true for  $\tan \theta$  and  $\tan \theta'$  and for the other four trigonometric functions. In all cases, the sign of the function value can be determined by the quadrant in which  $\theta$  lies.

#### Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle  $\theta$ :

- Determine the function value for the associated reference angle  $\theta'$ .
- Depending on the quadrant in which  $\theta$  lies, affix the appropriate sign to the function value.

Evaluate each trigonometric function.

a.  $\cos \frac{4\pi}{3}$     b.  $\tan(-210^\circ)$     c.  $\csc \frac{11\pi}{4}$

#### Solution

- a. Because  $\theta = 4\pi/3$  lies in Quadrant III, the reference angle is  $\theta' = (4\pi/3) - \pi = \pi/3$ , as shown in Figure 4.38. Moreover, the cosine is negative in Quadrant III, so

$$\cos \frac{4\pi}{3} = (-)\cos \frac{\pi}{3} = -\frac{1}{2}$$

- b. Because  $-210^\circ + 360^\circ = 150^\circ$ , it follows that  $-210^\circ$  is coterminal with the second-quadrant angle  $150^\circ$ . Therefore, the reference angle is  $\theta' = 180^\circ - 150^\circ = 30^\circ$ , as shown in Figure 4.39. Finally, because the tangent is negative in Quadrant II, you have

$$\tan(-210^\circ) = (-)\tan 30^\circ = -\frac{\sqrt{3}}{3}$$

- c. Because  $(11\pi/4) - 2\pi = 3\pi/4$ , it follows that  $11\pi/4$  is coterminal with the second-quadrant angle  $3\pi/4$ . Therefore, the reference angle is  $\theta' = \pi - (3\pi/4) = \pi/4$ , as shown in Figure 4.40. Because the cosecant is positive in Quadrant II, you have

$$\csc \frac{11\pi}{4} = (+)\csc \frac{\pi}{4} = \frac{1}{\sin(\pi/4)} = \sqrt{2}$$

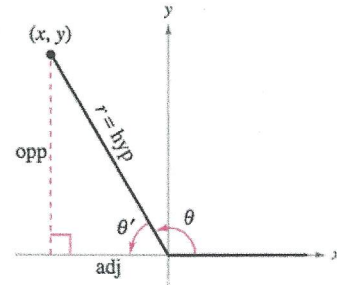


Figure 4.37

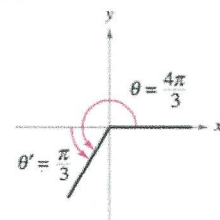


Figure 4.38

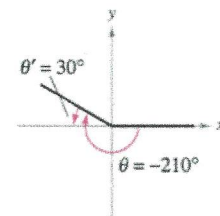


Figure 4.39

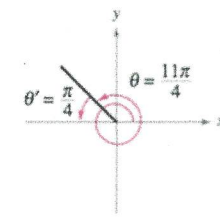


Figure 4.40

## Lesson 4.5 - Evaluating trig functions (part 2)

### Practice

In Exercises 53–64, evaluate the sine, cosine, and tangent of the angle without using a calculator.

53.  $225^\circ$   
 $\theta' = 45^\circ$  - Quadrant III

$$\sin = -\frac{\sqrt{2}}{2} \quad \cos = -\frac{\sqrt{2}}{2}$$

$$\tan = 1$$

54.  $300^\circ$   $\theta' = 60^\circ$  Quadrant IV

$$\sin = -\frac{\sqrt{3}}{2} \quad \cos = \frac{1}{2}$$

$$\tan = -\sqrt{3}$$

55.  $-750^\circ$   
 $\theta' = 30^\circ$  Quadrant IV

$$\sin = -\frac{1}{2} \quad \cos = \frac{\sqrt{3}}{2}$$

$$\tan = -\frac{\sqrt{3}}{3}$$

56.  $-495^\circ$   $\theta' = 45^\circ$  Quadrant III

$$\sin = -\frac{\sqrt{2}}{2} \quad \cos = -\frac{\sqrt{2}}{2}$$

$$\tan = 1$$

57.  $\frac{5\pi}{3}$   
 $\theta' = \frac{\pi}{3}$  Quadrant IV

$$\sin = -\frac{\sqrt{3}}{2} \quad \cos = \frac{1}{2}$$

$$\tan = -\sqrt{3}$$

58.  $\frac{3\pi}{4}$   $\theta' = \frac{\pi}{4}$  Quadrant II

$$\sin = \frac{\sqrt{2}}{2} \quad \cos = -\frac{\sqrt{2}}{2}$$

$$\tan = -1$$

59.  $-\frac{\pi}{6}$   
 $\theta' = \frac{\pi}{6}$  Quadrant IV

$$\sin = -\frac{1}{2} \quad \cos = \frac{\sqrt{3}}{2}$$

$$\tan = -1$$

60.  $-\frac{4\pi}{3}$   $\theta' = \frac{\pi}{3}$  Quadrant III

$$\sin = -\frac{\sqrt{3}}{2} \quad \cos = -\frac{1}{2}$$

$$\tan = \sqrt{3}$$

61.  $\frac{11\pi}{4}$   
 $\theta' = \frac{\pi}{4}$  Quadrant II

$$\sin = \frac{\sqrt{2}}{2} \quad \cos = -\frac{\sqrt{2}}{2}$$

$$\tan = -1$$

62.  $\frac{10\pi}{3}$   $\theta' = \frac{\pi}{3}$  Quadrant III

$$\sin = -\frac{\sqrt{3}}{2} \quad \cos = -\frac{1}{2}$$

$$\tan = \sqrt{3}$$

63.  $-\frac{17\pi}{6}$   
 $\theta' = \frac{\pi}{6}$  Quadrant IV

$$\sin = -\frac{1}{2} \quad \cos = \frac{\sqrt{3}}{2}$$

$$\tan = -\frac{\sqrt{3}}{3}$$

64.  $-\frac{20\pi}{3}$   $\theta' = \frac{\pi}{3}$  Quadrant III

$$\sin = -\frac{\sqrt{3}}{2} \quad \cos = -\frac{1}{2}$$

$$\tan = \sqrt{3}$$