Lesson 4.5 - Evaluating trig functions (part 2)

Learning Objectives: SWBAT

1. Use reference angles to evaluate trig functions for a given angle

Making a connection

- In lesson 4.3, we learned to evaluate trig functions for a given angle if we were also given specific information about the quadrant it was in.
- In this lesson, we will use reference angles to determine the quadrant the angle is in before evaluating.

To see how a reference angle is used to evaluate a trigonometric function, consider the point (x, y) on the terminal side of θ, as shown in Figure 4.37. By definition, you know that

$$\sin\theta = \frac{y}{r} \quad \text{and} \quad \tan\theta = \frac{y}{x}.$$

For the right triangle with acute angle θ' and sides of lengths $|x|$ and $|y|$, you have

$$\sin\theta' = \frac{\text{opp}}{\text{hyp}} = \frac{|y|}{r}$$

and

$$\tan\theta' = \frac{\text{opp}}{\text{adj}} = \frac{|y|}{|x|}.$$

So, it follows that $\sin\theta$ and $\sin\theta'$ are equal, *except possibly in sign*. The same is true for $\tan\theta$ and $\tan\theta'$ and for the other four trigonometric functions. In all cases, the sign of the function value can be determined by the quadrant in which θ lies.

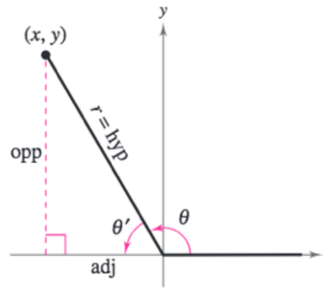

Figure 4.37

Evaluating Trigonometric Functions of Any Angle

To find the value of a trigonometric function of any angle θ:

1. Determine the function value for the associated reference angle θ'.
2. Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Evaluate each trigonometric function.

a. $\cos\frac{4\pi}{3}$ **b.** $\tan(-210°)$ **c.** $\csc\frac{11\pi}{4}$

Solution

a. Because $\theta = 4\pi/3$ lies in Quadrant III, the reference angle is $\theta' = (4\pi/3) - \pi = \pi/3$, as shown in Figure 4.38. Moreover, the cosine is negative in Quadrant III, so

$$\cos\frac{4\pi}{3} = (-)\cos\frac{\pi}{3} = -\frac{1}{2}.$$

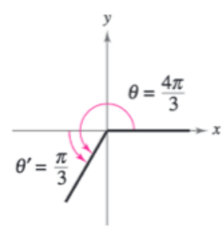

Figure 4.38

b. Because $-210° + 360° = 150°$, it follows that $-210°$ is coterminal with the second-quadrant angle $150°$. Therefore, the reference angle is $\theta' = 180° - 150° = 30°$, as shown in Figure 4.39. Finally, because the tangent is negative in Quadrant II, you have

$$\tan(-210°) = (-)\tan 30° = -\frac{\sqrt{3}}{3}.$$

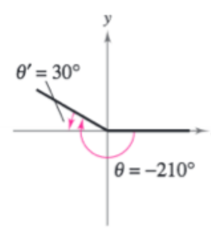

Figure 4.39

c. Because $(11\pi/4) - 2\pi = 3\pi/4$, it follows that $11\pi/4$ is coterminal with the second-quadrant angle $3\pi/4$. Therefore, the reference angle is $\theta' = \pi - (3\pi/4) = \pi/4$, as shown in Figure 4.40. Because the cosecant is positive in Quadrant II, you have

$$\csc\frac{11\pi}{4} = (+)\csc\frac{\pi}{4} = \frac{1}{\sin(\pi/4)} = \sqrt{2}.$$

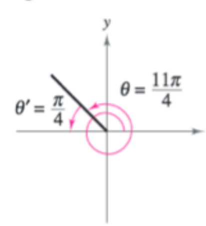

Figure 4.40

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Practice

In Exercises 53–64, evaluate the sine, cosine, and tangent of the angle without using a calculator. sketch the reference angle

53. $225°$

54. $300°$

55. $-750°$

56. $-495°$

57. $\frac{5\pi}{3}$

58. $\frac{3\pi}{4}$

59. $-\frac{\pi}{6}$

60. $-\frac{4\pi}{3}$

61. $\frac{11\pi}{4}$

62. $\frac{10\pi}{3}$

63. $-\frac{17\pi}{6}$

64. $-\frac{20\pi}{3}$